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operator $(-)^{\vee}$

An encoding

Conclusions 00

On the semantics of disjunctive logic programs

— Part II —

Thanos Tsouanas

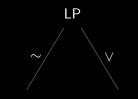
 $\texttt{UFRN} \leftarrow \texttt{ENS} \texttt{de} \texttt{Lyon}$

Universidade Federal do Rio Grande do Norte, Natal February 27th, 2015

Previously	≡	Negation	Truth value spaces	ASF	The operator $(-)^{\vee}$	An encoding	Conclusions
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Previously on Semantics of Logic Programs...

00000000 0 0000000 00 00 00 00000 00 00	Previously	≡	Negation	Truth value spaces	ASF	The operator $(-)^{\vee}$	An encoding	Conclusions
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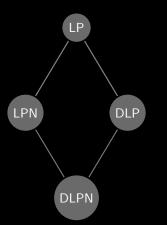
Previously	=	Negation	Truth value spaces	ASF	The operator $(-)^{\vee}$	An encoding	Conclusions
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Example of LP $\left\{ \begin{array}{l} \texttt{sleeps} \leftarrow \texttt{tired} \\ \texttt{works} \leftarrow \texttt{rested} \\ \texttt{eats} \leftarrow \texttt{rested} \ \texttt{, hungry} \\ \texttt{rested} \leftarrow \end{array} \right\}$

Queries

Previously	≡	Negation	Truth value spaces	ASF	The operator $(-)^{ee}$	An encoding	Conclusions
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Semantics Model-theoretic



Model-theoretic semantics

LP: least Herbrand model LPN: well-founded model DLP: minimal models DLPN: ∞-valued minimal models



$$\mathcal{P} \coloneqq \left\{ \begin{array}{l} \mathbf{a} \leftarrow \mathbf{e} \\ \mathbf{b} \leftarrow \mathbf{d} \\ \mathbf{b} \leftarrow \mathbf{e} \\ \mathbf{e} \leftarrow \\ \mathbf{f} \leftarrow \end{array} \right\} \quad \left| \begin{array}{l} \mathsf{goal} : & \leftarrow \underline{p} \\ \mathsf{B}_0 : & \mathbf{p} \leftarrow \mathbf{a} \ , \underline{t} \\ \mathsf{B}_1 : & \mathbf{b} \leftarrow \mathbf{d} \\ \mathsf{B}_1 : & \mathbf{b} \leftarrow \mathbf{d} \end{array} \right|$$

Believer lost! ~



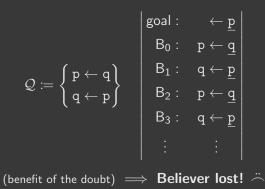
$$\mathcal{P} := \begin{cases} p \leftarrow a \ , b \\ p \leftarrow c \\ a \leftarrow e \\ b \leftarrow d \\ b \leftarrow e \\ e \leftarrow \\ f \leftarrow \end{cases} \end{cases} \left. \begin{array}{c} \text{goal} : & \leftarrow \underline{p} \\ B_0 : & p \leftarrow \underline{a} \ , b \\ B_1 : & a \leftarrow \underline{e} \\ B_2 : & e \leftarrow \emptyset \end{array} \right.$$

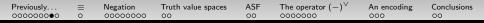
Example (2)

Believer wins! Ü

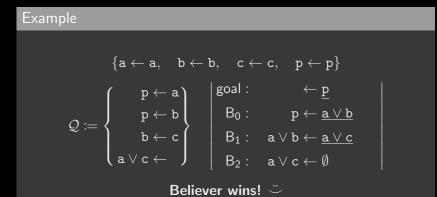


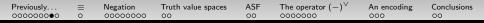
Example (3)



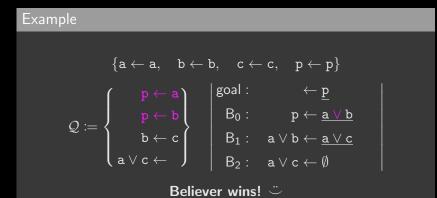


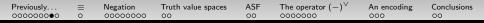
DLP game = LP game + combo + implicit



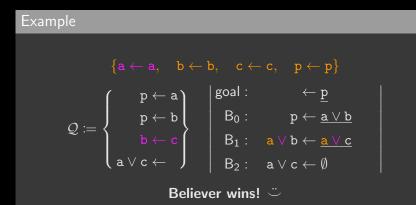


DLP game = LP game + combo + implicit





DLP game = LP game + combo + implicit



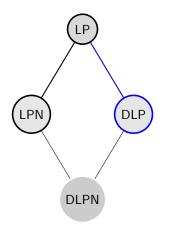
Negation 0000000 Truth value spa

SF The ope 0 00000

oerator (−)[∨] 000 An encoding

Conclusions

Where we are



Model-theoretic semantics

- LP: least Herbrand model
- LPN: well-founded model
- **DLP:** minimal models
- **DLPN:** ∞ -valued minimal models

Game semantics

- LP: Di Cosmo, Loddo & Nicolet (1998)
- LPN: Rondogiannis & Wadge (2005)
- DLP: me (2013)
- DLPN: ?

Previously	≡	Negation	Truth value spaces	ASF	The operator $(-)^{\vee}$	An encoding	Conclusions
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- 1 Negation as failure
- 2 Truth value spaces
- 3 An abstract semantic framework
- 4 The semantic operator $(-)^{\vee}$
- 5 An encoding from DLP into LP
- 6 Conclusions

Previously		Negation	Truth value spaces	ASF	The operator $(-)^{\vee}$	An encoding	Conclusions
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The DLP and LPN games

The DLP game

$\mathsf{DLP}\ \mathsf{game} = \mathsf{LP}\ \mathsf{game} + \mathsf{combo} + \mathsf{implicit}$

The LPN game

LPN game = LP game + rôle-switch

Previously		Negation	Truth value spaces	ASF	The operator $(-)^{\vee}$	An encoding	Conclusions
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The DLP and LPN games

The DLP game

 $\mathsf{DLP}\ \mathsf{game} = \mathsf{LP}\ \mathsf{game} + \mathsf{combo} + \mathsf{implicit}$

The LPN game

LPN game = LP game + rôle-switch

Previously... ≡ 000000000 0 Negation ○●○○○○○○ n value space

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Conclusions 00

A game semantics for LPN

The LPN game

Opponent vs Player

Whenever a doubter doubts a negated atom ~p, the rôles of the players switch: the believer becomes the doubter, doubting p.

This implies draws!

Previously... \equiv NegationTruth value spacesASFThe operator $(-)^{\vee}$ An encoding000000000000000000000000

A game semantics for LPN

The LPN game

- Opponent vs Player
- Whenever a doubter doubts a negated atom ~p, the rôles of the players switch: the believer becomes the doubter, doubting p.

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A game semantics for LPN

The LPN game

- Opponent vs Player
- Whenever a doubter doubts a negated atom ~p, the rôles of the players switch: the believer becomes the doubter, doubting p.
- This implies draws!

Pr

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The operator $(-)^{\vee}$ 0000000

An encoding

The LPN game

Example plays (1)

$$\mathcal{P} \coloneqq \left\{ \begin{array}{l} \mathbf{p} \leftarrow \\ \mathbf{q} \leftarrow \sim \mathbf{p} \\ \mathbf{r} \leftarrow \sim \mathbf{q} \end{array} \right\},$$
$$\pi_1 \coloneqq \left| \begin{array}{c} \operatorname{goal} : & \leftarrow \mathbf{q} \\ \hline \mathbf{P}_0 : & \mathbf{q} \leftarrow \underline{\sim \mathbf{p}} \\ \hline \mathbf{O}_2 : & \mathbf{p} \leftarrow \end{array} \right|, \qquad \pi_2 \coloneqq \left| \begin{array}{c} \operatorname{goal} : & \leftarrow \mathbf{r} \\ \hline \mathbf{P}_0 : & \mathbf{r} \leftarrow \underline{\sim \mathbf{q}} \\ \hline \mathbf{O}_2 : & \mathbf{q} \leftarrow \underline{\sim \mathbf{p}} \\ \hline \mathbf{P}_3 : & \mathbf{p} \leftarrow \end{array} \right|$$

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An encoding

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Conclusions 00

The LPN game

Example plays (1)

$$\begin{split} \mathcal{P} &\coloneqq \left\{ \begin{matrix} \mathbf{p} \leftarrow \\ \mathbf{q} \leftarrow \sim \mathbf{p} \\ \mathbf{r} \leftarrow \sim \mathbf{q} \end{matrix} \right\}, \\ \pi_1 &\coloneqq \left| \begin{matrix} \mathsf{goal} : & \leftarrow \mathbf{q} \\ \hline \mathbf{P}_0 : & \mathbf{q} \leftarrow \underline{\sim \mathbf{p}} \\ \hline \mathbf{O}_2 : & \mathbf{p} \leftarrow \end{matrix} \right|, \qquad \pi_2 &\coloneqq \left| \begin{matrix} \mathsf{goal} : & \leftarrow \mathbf{r} \\ \hline \mathbf{P}_0 : & \mathbf{r} \leftarrow \underline{\sim \mathbf{q}} \\ \hline \mathbf{O}_2 : & \mathbf{q} \leftarrow \underline{\sim \mathbf{p}} \\ \hline \mathbf{P}_3 : & \mathbf{p} \leftarrow \end{matrix} \right| \end{split}$$

Previously... ≡ 000000000 0 Negation 000●0000 Truth value spaces

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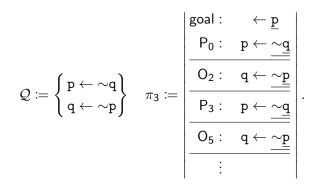
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Conclusions 00

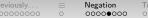
The LPN game

Example plays (2)

How about this infamous program of LPN:



Here no player is able to secure the believer rôle for themselves, so this play is won by neither of them: *the outcome is a tie*.



Truth	value	sp
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The operator $(-)^{\vee}$

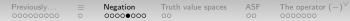
An encoding 000 Conclusions 00

Truth values for NAF



Negation

\sim F = T	\sim U = U	$\sim \mathbf{T} = \mathbf{F}$



 $\sim \mathbf{F} = \mathbf{T}$

 $\sim T = F$

An encoding

Truth values for NAF



 $\sim U = U$

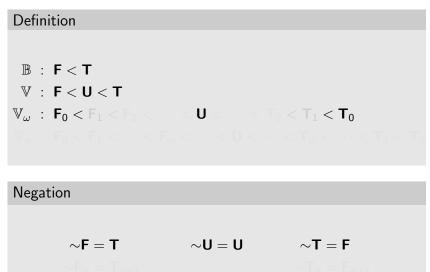


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Conclusions

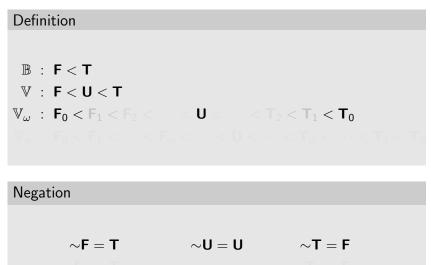




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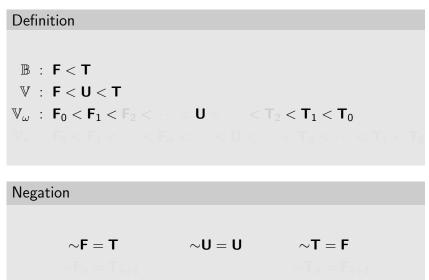


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Conclusions 00



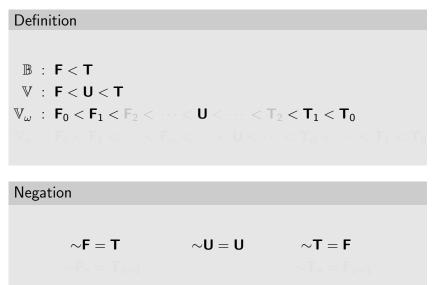


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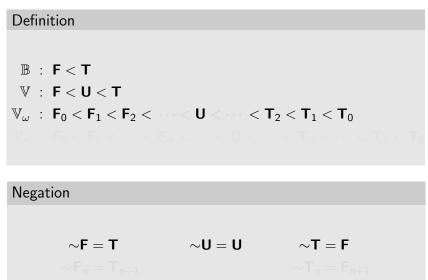
Conclusions





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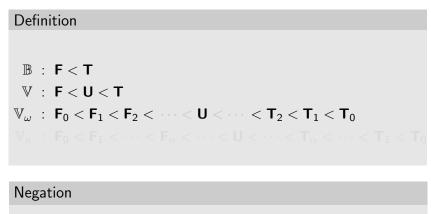




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Truth values for NAF



 $\sim \mathbf{F} = \mathbf{T}$ $\sim U = U$ $\sim T = F$



spaces ASI

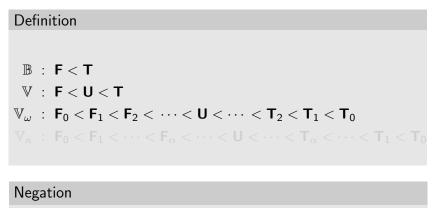
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tor (−) [∨] ,

An encoding

Conclusions 00

Truth values for NAF



$\sim \mathbf{F} = \mathbf{T} \qquad \sim \mathbf{U} = \mathbf{U} \qquad \sim \mathbf{T} = \mathbf{F}$ $\sim \mathbf{F}_n = \mathbf{T}_{n+1} \qquad \qquad \sim \mathbf{T}_n = \mathbf{F}_{n+1}$

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ASF The operator $(-)^{\vee}$ An encoding

Truth values for NAF



Negation

\sim F = T	\sim U = U	\sim T = F
$\sim \mathbf{F}_n = \mathbf{T}_{n+1}$		$\sim \mathbf{T}_n = \mathbf{F}_{n+1}$

Previously		Negation	Truth value spaces	ASF	The o
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operator $(-)^{\vee}$ An encoding Conclusions

Truth values for NAF

Definition \mathbb{B} : $\mathbf{F} < \mathbf{T}$ \mathbb{V} : $\mathbf{F} < \mathbf{U} < \mathbf{T}$ \mathbb{V}_{ω} : $\mathbf{F}_0 < \mathbf{F}_1 < \mathbf{F}_2 < \cdots < \mathbf{U} < \cdots < \mathbf{T}_2 < \mathbf{T}_1 < \mathbf{T}_0$ \mathbb{V}_{κ} : $\mathbf{F}_0 < \mathbf{F}_1 < \cdots < \mathbf{F}_{\alpha} < \cdots < \mathbf{U} < \cdots < \mathbf{T}_{\alpha} < \cdots < \mathbf{T}_1 < \mathbf{T}_0$

Negation

\sim F = T	\sim U = U	\sim T = F
$\sim \mathbf{F}_n = \mathbf{T}_{n+1}$		$\sim \mathbf{T}_n = \mathbf{F}_{n+1}$

Previously		Negation	Truth value spaces	ASF	The operator $(-)^{\vee}$	An encoding	Conclusions
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Payoff function

Definition (Payoff)

Let π be a play in some LPN game. Then the **payoff** functions Φ_{ω} and Φ are defined by:

$$\Phi_{\omega}(\pi) \triangleq \begin{cases} \mathbf{T}_{n}, & \text{if Player wins in } \pi, \\ \mathbf{F}_{n}, & \text{if Player loses in } \pi, \\ \mathbf{U}, & \text{otherwise}, \end{cases}$$

where *n* is the number of rôle-switching moves played in π ; and

$$\Phi \triangleq collapse \circ \Phi_{\omega},$$

where *collapse* is the "subscript-removing" function.

Example payoffs

$$\underbrace{\begin{vmatrix} \mathbf{goal} : & \leftarrow \mathbf{\underline{q}} \\ \underline{P_0 : & \mathbf{q} \leftarrow \underline{-p} \\ 0_2 : & \mathbf{p} \leftarrow \\ \pi_1 \end{matrix}}_{\pi_1} \underbrace{\begin{vmatrix} \mathbf{goal} : & \leftarrow \mathbf{\underline{r}} \\ \underline{P_0 : & \mathbf{r} \leftarrow \underline{-q} \\ 0_2 : & \mathbf{q} \leftarrow \underline{-p} \\ \hline \mathbf{P_3 : & \mathbf{p} \leftarrow \\ \pi_2 \end{matrix}}_{\pi_2} \underbrace{\begin{vmatrix} \mathbf{goal} : & \leftarrow \mathbf{\underline{p}} \\ \underline{P_0 : & \mathbf{p} \leftarrow \underline{-q} \\ 0_2 : & \mathbf{q} \leftarrow \underline{-p} \\ \hline \mathbf{Q}_2 : & \mathbf{q} \leftarrow \underline{-p} \\ \hline \mathbf{P_3 : & \mathbf{p} \leftarrow \underline{-q} \\ \vdots \\ \pi_3 \end{aligned}}$$

The corresponding payoffs are:

$$\begin{split} \Phi_{\omega}(\pi_1) &= \mathsf{F}_1, \qquad \Phi_{\omega}(\pi_2) = \mathsf{T}_2, \qquad \Phi_{\omega}(\pi_3) = \mathsf{U}. \\ \Phi(\pi_1) &= \mathsf{F}, \qquad \Phi(\pi_2) = \mathsf{T}, \qquad \Phi(\pi_3) = \mathsf{U}. \end{split}$$

Example payoffs

$$\underbrace{\begin{vmatrix} goal : & \leftarrow \underline{q} \\ \hline P_0 : & q \leftarrow \underline{\neg p} \\ \hline O_2 : & p \leftarrow \\ \hline \pi_1 \end{matrix}}_{\pi_1} \underbrace{\begin{vmatrix} goal : & \leftarrow \underline{r} \\ \hline P_0 : & r \leftarrow \underline{\neg q} \\ \hline O_2 : & q \leftarrow \underline{\neg p} \\ \hline P_3 : & p \leftarrow \\ \hline \pi_2 \end{matrix}}_{\pi_2} \underbrace{\begin{vmatrix} goal : & \leftarrow \underline{p} \\ \hline P_0 : & p \leftarrow \underline{\neg q} \\ \hline O_2 : & q \leftarrow \underline{\neg p} \\ \hline O_2 : & q \leftarrow \underline{\neg p} \\ \hline \hline P_3 : & p \leftarrow \underline{\neg q} \\ \hline \vdots \\ \hline \end{array}}_{\pi_3}$$

The corresponding payoffs are:

$$\begin{aligned} \Phi_{\omega}(\pi_1) &= \mathbf{F}_1, & \Phi_{\omega}(\pi_2) &= \mathbf{T}_2, & \Phi_{\omega}(\pi_3) &= \mathbf{U}. \\ \Phi(\pi_1) &= \mathbf{F}, & \Phi(\pi_2) &= \mathbf{T}, & \Phi(\pi_3) &= \mathbf{U}. \end{aligned}$$

Previously		Negation	Truth value spaces	ASF	The operator $(-)^{\vee}$	An encoding	Conclusions
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How do we get a semantics out of LPNG?

3-valued

The value of q is... $\begin{cases} \textbf{T}, & \text{if there is a winning strategy} \\ \textbf{U}, & \text{else, if there is a non-losing strategy} \\ \textbf{F}, & \text{otherwise.} \end{cases}$

```
\infty-valued
The value of q is...
\sup \Big\{ \inf \{ \Phi_{\omega}(\pi) \mid \pi \in \sigma \} \mid \sigma \text{ is a strategy for } q \Big\}.
```

Previously		Negation	Truth value spaces	ASF	The operator $(-)^{\vee}$	An encoding	Conclusions
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How do we get a semantics out of LPNG?

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∞ -valued

The value of q is. . .

$$\sup\Big\{\inf\left\{\Phi_\omega(\pi)\ \mid\ \pi\in\sigma\right\}\ \Big|\ \sigma \text{ is a strategy for }q\Big\}\,.$$



Truth value spaces

Defininition

A truth value space ${\cal V}$ is a completely distributive Heyting algebra with an additional unary operator \sim .

Weaponry

$$\begin{array}{ccc} \top, & \bot, & x < y, & x > y \\ x \land y, & x \lor y, & \bigwedge S, & \bigvee S, & \sim x, & x \Rightarrow y \end{array}$$

... and they all behave!

Previously 000000000	= 0	Negation 00000000	Truth value spaces ○●	ASF oo	The operator $(-)^{\vee}$ 000000	An encoding 000	Conclusions 00			
Truth value spaces										
Exan	nple	s of truth	value spaces							

$$\mathbb{B}, \mathbb{V}, \mathbb{V}_{\omega}, \mathbb{V}_{\kappa}, \ldots$$

The total order of the bounded set \mathbb{V}_{κ} determines:

$$\begin{array}{ll} x \lor y &= \max\left\{x,y\right\}, \\ x \land y &= \min\left\{x,y\right\}, \end{array} \quad \text{and} \quad x \Rightarrow y = \begin{cases} \top & \text{if } x \leq y, \\ y & \text{otherwise.} \end{cases}$$

It remains to define the operator \sim :

$$\sim x \triangleq \begin{cases} \mathbf{T}_{\alpha+1} & \text{if } x = \mathbf{F}_{\alpha}, \\ \mathbf{F}_{\alpha+1} & \text{if } x = \mathbf{T}_{\alpha}, \\ \mathbf{U} & \text{if } x = \mathbf{U}. \end{cases}$$

Previously		Negation	Truth value spaces	ASF	The operator $(-)^{\vee}$	An encoding	Conclusions
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An abstract semantic framework ASF (1)

Definition

Let:

- L : a logic programming language,
- ${\mathcal M}\,$: a set of "meanings",
 - $\ensuremath{\mathcal{V}}$: a truth value space.

Then:

 $\begin{array}{lll} \mathcal{M}\text{-semantics for } \mathcal{L} & \mathbf{m} \ : \ \mathbf{P}_L \to \mathcal{M}; \\ \mathcal{V}\text{-answer function for } \mathcal{M} & \mathbf{a} \ : \ \mathcal{M} \to \mathbf{Q}_L \to \mathcal{V}; \\ \mathcal{V}\text{-system for } \mathcal{L} & \mathbf{s} \ : \ \mathbf{P}_L \to \mathbf{Q}_L \to \mathcal{V}; \\ \text{semantics for } \mathcal{L} & (\mathbf{m}, \mathbf{a}) \ \rightsquigarrow \ \mathbf{a} \circ \mathbf{m} \ : \ \mathbf{P}_L \to \mathbf{Q}_L \to \mathcal{V}. \end{array}$

 $\begin{array}{cccc} Previously... & \equiv & Negation & Truth value spaces & ASF & The operator <math>(-)^{\vee} & An \mbox{ encoding } Conclusions & c$

An abstract semantic framework ASF (2)

Defining the notions of...

- truth value space \mathcal{V} (e.g., \mathbb{B} , \mathbb{V}_{κ} , ...)
- semantics of a language L as objects of study

 $s : \mathbf{P}_L \times \mathbf{Q}_L \rightarrow \mathcal{V};$

• equivalence of semantics (pprox);

```
■ refinement of semantics (<);</p>
```

. . . .

An abstract semantic framework ASF (2)

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- truth value space \mathcal{V} (e.g., \mathbb{B} , \mathbb{V}_{κ} , ...)
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```
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An abstract semantic framework ASF (2)

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equivalence of semantics (≈);
 refinement of semantics (⊲);

An abstract semantic framework ASF (2)

Defining the notions of...

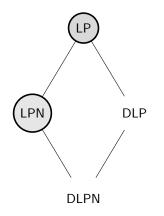
- truth value space \mathcal{V} (e.g., \mathbb{B} , \mathbb{V}_{κ} , ...)
- semantics of a language *L* as objects of study

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- equivalence of semantics (\approx);
- refinement of semantics (\lhd);
- ...

The operator $(-)^{\vee}$

The semantic operator $(-)^{\vee}$



Transforming semantics

- Start with any semantics s for a non-disjunctive language.
- Apply the operator to get a new

Previously...

Negation 0000000 th value space

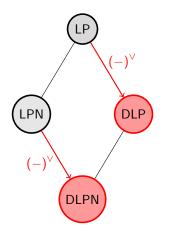
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The operator $(-)^{\vee}$ •000000

An encoding

Conclusions

The semantic operator $(-)^{\vee}$



Transforming semantics

- Start with any semantics s for a non-disjunctive language.
- Apply the operator to get a new semantics (s)[∨] for the corresponding disjunctive language.

 $\begin{array}{rrrr} \mbox{Previously...} & \equiv & \mbox{Negation} & \mbox{Truth value spaces} & \mbox{ASF} & \mbox{The operator (-)}^V & \mbox{An encoding} & \mbox{Conclusions} \\ \mbox{occocccc} & \mbox{occ} & \mbox{occc} & \mbox{occ} & \mbox{occ} & \m$

Properties of
$$(-)^{\vee}$$

Preservation properties

The operator respects equivalences and refinements:

$$egin{array}{lll} s_1pprox s_2 \implies (s_1)^eepprox (s_2)^ee\ s_1 \lhd s_2 \implies (s_1)^ee \lhd (s_2)^ee\ \end{array}$$

Expected outcomes

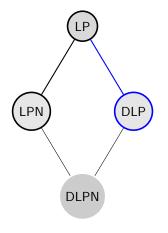
The operator yields the expected results for the standard semantics:

(Least Herbrand Model)^{\vee} \approx Minimal Models (Well-Founded Model)^{\vee} \approx ∞ -valued Minimal Models

The operator $(-)^{\vee}$

An encoding

Applications of $(-)^{\vee}$ on game semantics



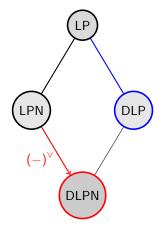
Game semantics

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- **DLP:** me (2013)
- DLPN: ?

The operator $(-)^{\vee}$

An encoding

Applications of $(-)^{\vee}$ on game semantics



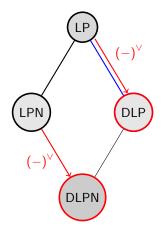
Game semantics

LP: Di Cosmo, Loddo & Nicolet (1998) LPN: Rondogiannis & Wadge (2005) **DLP:** me (2013) DLPN: me (2014)

The operator $(-)^{\vee}$

An encoding

Applications of $(-)^{\vee}$ on game semantics



Game semantics

LP: Di Cosmo, Loddo & Nicolet (1998) LPN: Rondogiannis & Wadge (2005) **DLP:** me (2013) \approx me (2014) DLPN: me (2014)



Consider the disjunctive program

$$\mathcal{D}\coloneqq \left\{ egin{array}{ll} \mathtt{s} \ ee \ \mathtt{t} \ \leftarrow \mathtt{p} \ , \mathtt{b} \ , \mathtt{c} \ \mathtt{a} \ ee \ \mathtt{b} \ \leftarrow & \ \mathtt{p} \ \leftarrow \mathtt{a} \ \mathtt{p} \ \leftarrow \mathtt{a} \ \mathtt{p} \ \leftarrow \mathtt{b} \ , \mathtt{d} \ , \mathtt{f} \ \mathtt{b} \ ee \ \mathtt{c} \ \leftarrow & \end{array}
ight\}$$

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The set $\mathrm{D}(\mathcal{D})$ of its definite instantiations is

 $\mathrm{D}(\mathcal{D}) = \{ \mathbb{P}_{0}, \mathbb{P}_{0}, \mathbb{P}_{0}, \mathbb{P}_{0}, \mathbb{P}_{0}, \mathbb{P}_{0}, \mathbb{P}_{0}, \mathbb{P}_{0} \}.$



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Consider the disjunctive program

$$\mathcal{D} := \left\{ \begin{array}{c} (\textbf{s} \lor \textbf{t} \leftarrow \textbf{p} , \textbf{b} , \textbf{c} \\ (\textbf{a} \lor \textbf{b} \leftarrow \textbf{c} \\ \textbf{p} \leftarrow \textbf{a} \\ (\textbf{p} \leftarrow \textbf{b} , \textbf{d} , \textbf{f} \\ \textbf{b} \lor \textbf{c} \leftarrow \textbf{c} \end{array} \right\}$$

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$$\mathcal{D} := \begin{cases} (\textbf{s} \lor \textbf{t} \leftarrow \textbf{p}, \textbf{b}, \textbf{c}) \\ (\textbf{a} \lor \textbf{b} \leftarrow \textbf{c}) \\ (\textbf{p} \leftarrow \textbf{a}) \\ (\textbf{p} \leftarrow \textbf{b}, \textbf{d}, \textbf{f}) \\ (\textbf{b} \lor \textbf{c} \leftarrow \textbf{c}) \end{cases}$$

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 $\begin{array}{c|cccc} Previously... & \equiv & Negation & Truth value spaces & ASF & The operator <math>(-)^V & An \ encoding & Conclusions & OOOOOOOO & OOO &$

Definition of $(-)^{\vee}$ for a semantics **s** of LP

$$\begin{array}{ll} \text{if} & \textbf{s} \ : \ \textbf{P}_{\mathsf{LP}} \times \textbf{Q}_{\mathsf{LP}} \to \mathcal{V}, \\ \text{then} & (\textbf{s})^{\vee} \ : \ \textbf{P}_{\mathsf{DLP}} \times \textbf{Q}_{\mathsf{DLP}} \to \mathcal{V}, \\ \text{is defined by} & (\textbf{s})^{\vee}(\mathcal{D}, \mathcal{G}) \triangleq \end{array}$$

```
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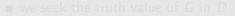
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$$\mathrm{D}(\mathcal{D}) = \{\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \mathcal{P}_4, \mathcal{P}_5, \mathcal{P}_6, \mathcal{P}_7, \mathcal{P}_8\}.$$

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An example of
$$(-)^{\vee}$$
 in use

$$\mathcal{D} := \left\{ \begin{array}{ll} \texttt{s} \lor (\texttt{t} \leftarrow \texttt{p} \ , \texttt{b} \ , \texttt{c} \\ \texttt{a} \lor (\texttt{b} \leftarrow \texttt{c} \\ \texttt{p} \leftarrow \texttt{a} \\ \texttt{p} \leftarrow \texttt{b} \ , \texttt{d} \ , \texttt{f} \\ \texttt{b} \lor (\texttt{c} \leftarrow \texttt{c} \end{array} \right\} \qquad \left| \begin{array}{ll} \texttt{goal:} & \leftarrow \texttt{p} \lor \texttt{s} \lor \texttt{t} \\ \texttt{D}_0: \\ \end{array} \right.$$

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$$\mathcal{P}_4 := \left\{ \begin{array}{ccc} (\texttt{t} \leftarrow \texttt{p} \ , \texttt{b} \ , \texttt{c} \\ (\texttt{b} \leftarrow \texttt{b} \\ \texttt{p} \leftarrow \texttt{a} \\ \texttt{p} \leftarrow \texttt{b} \ , \texttt{d} \ , \texttt{f} \\ \texttt{c} \leftarrow \end{array} \right\} \qquad \left| \begin{array}{c} \texttt{goal} : & \leftarrow \texttt{p} \ \lor \texttt{s} \ \lor \texttt{t} \\ \texttt{D}_0 : & \mathcal{P}_4 \\ \texttt{D}_0 : & \mathcal{P}_4 \end{array} \right.$$

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The operator $(-)^{\vee}$ Ar 000000

An encoding

Conclusions

What is $(LPG)^{\vee}$?

An example of $(-)^{\vee}$ in use

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$$\mathcal{P}_{4} := \left\{ \begin{array}{cccc} \mathbf{t} \leftarrow \mathbf{p} \ , \ \mathbf{b} \ , \ \mathbf{c} \\ \mathbf{b} \leftarrow \\ \mathbf{p} \leftarrow \mathbf{a} \\ \mathbf{p} \leftarrow \mathbf{b} \ , \ \mathbf{d} \ , \ \mathbf{f} \\ \mathbf{c} \leftarrow \end{array} \right\} \qquad \left| \begin{array}{cccc} \operatorname{goal} : & \leftarrow \ \mathbf{p} \ \lor \ \mathbf{s} \ \lor (\mathbf{t}) \\ \mathbf{D}_{0} : & \mathcal{P}_{4} \\ \mathbf{B}_{0} : & \mathbf{t} \\ \end{array} \right| \\ \mathbf{B}_{0} : & \mathbf{t} \end{array} \right|$$

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$$\mathcal{P}_4 := \left\{ \begin{array}{cccc} \texttt{t} \leftarrow \texttt{p} \ , \texttt{b} \ , \texttt{c} \\ \texttt{b} \leftarrow & \\ \texttt{p} \leftarrow \texttt{a} \\ \texttt{p} \leftarrow \texttt{b} \ , \texttt{d} \ , \texttt{f} \\ \texttt{c} \leftarrow & \end{array} \right\} \qquad \left| \begin{array}{cccc} \texttt{goal} : & \leftarrow \texttt{p} \ \lor \ \texttt{s} \ \lor \ \texttt{t} \\ \texttt{D}_0 : & \mathcal{P}_4 \\ \texttt{B}_0 : & \texttt{t} \\ \texttt{goal} : & \leftarrow \texttt{t} \end{array} \right.$$

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$$\mathcal{P}_4 := \left\{ \begin{array}{cccc} t \leftarrow p \ , b \ , c \\ b \leftarrow \\ p \leftarrow a \\ p \leftarrow b \ , d \ , f \\ c \leftarrow \end{array} \right\} \qquad \left| \begin{array}{cccc} \text{goal} : & \leftarrow p \ \lor \ s \ \lor \ t \\ D_0 : & \mathcal{P}_4 \\ B_0 : & t \\ \text{goal} : & \leftarrow \underline{t} \end{array} \right.$$

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An example of $(-)^{\vee}$ in use

$$\mathcal{P}_4 \coloneqq \left\{ \begin{array}{ccc} t \leftarrow p \ , b \ , c \\ b \leftarrow \\ p \leftarrow a \\ p \leftarrow b \ , d \ , f \\ c \leftarrow \end{array} \right\} \qquad \left| \begin{array}{c} \text{goal} : & \leftarrow p \ \lor \ s \ \lor \ t \\ D_0 : \quad \mathcal{P}_4 \\ B_0 : \quad t \\ \text{goal} : & \leftarrow \underline{t} \\ B_1 : \quad t \leftarrow p \ , b \ , c \end{array} \right.$$

$$D(\mathcal{D}) = \{\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \mathcal{P}_4, \mathcal{P}_5, \mathcal{P}_6, \mathcal{P}_7, \mathcal{P}_8\}.$$

■ Negation

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The operator $(-)^{\vee}$ 0000000 An encoding

Conclusions

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$$\mathcal{P}_4 := \left\{ \begin{array}{cccc} \mathbf{t} \leftarrow \mathbf{p} \ , \ \mathbf{b} \ , \ \mathbf{c} \\ \mathbf{b} \leftarrow & \\ \mathbf{p} \leftarrow \mathbf{a} \\ \mathbf{p} \leftarrow \mathbf{b} \ , \ \mathbf{d} \ , \ \mathbf{f} \\ \mathbf{c} \leftarrow & \end{array} \right\} \qquad \left| \begin{array}{cccc} \operatorname{goal}: & \leftarrow \ \mathbf{p} \ \lor \ \mathbf{s} \ \lor \ \mathbf{t} \\ \mathbf{D}_0: & \mathcal{P}_4 \\ \mathbf{B}_0: & \mathbf{t} \\ \operatorname{goal}: & \leftarrow \ \mathbf{t} \\ \mathbf{B}_1: & \mathbf{t} \leftarrow \mathbf{p} \ , \ \mathbf{b} \ , \ \mathbf{c} \end{array} \right|$$

 $\mathrm{D}(\mathcal{D}) = \{\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \frac{\mathcal{P}_4}{\mathcal{P}_5}, \mathcal{P}_6, \mathcal{P}_7, \mathcal{P}_8\}.$

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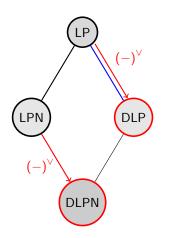
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The operator $(-)^{\vee}$

An encoding

Applications of $(-)^{\vee}$



Model-theoretic semantics

- LP: least Herbrand model
- LPN: well-founded model
- **DLP:** minimal models
- **DLPN:** ∞-valued minimal models

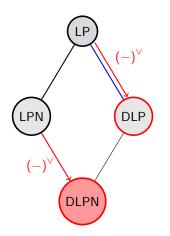
Game semantics

LP: Di Cosmo, Loddo & Nicolet (1998) LPN: Rondogiannis & Wadge (2005) **DLP:** me (2013) \approx me (2014) **DLPN:** me (2014)

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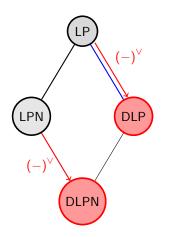
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Encoding of DLP into LP

The idea: suppose that we are given...

a finite DLP program \mathcal{D} and a DLP goal $\mathcal{G} := \{g_1, \ldots, g_m\}$.

We can encode both DLP objects DLP into objects of LP with:

 $\textit{encode} \ : \ \textbf{P}_{\mathsf{DLP}} \times \textbf{Q}_{\mathsf{DLP}} \rightarrow \textbf{P}_{\mathsf{LP}} \times \textbf{Q}_{\mathsf{LP}},$

so that if $encode(\mathcal{D}, G) = (\mathcal{P}, g)$, we can use the LP game on \mathcal{P} with the goal g to obtain an answer for the initial DLP goal G, w.r.t. the initial program \mathcal{D} .



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Given $D(\mathcal{D}) := \{\mathcal{P}_1, \dots, \mathcal{P}_n\}$ and $G := g_1 \vee \cdots \vee g_m$, define: $encode(\mathcal{D}, G) \triangleq (\mathcal{P}, g)$ $\mathcal{P} := \mathcal{P}_1 \uplus \cdots \uplus \mathcal{P}_n \cup restrictors(\mathcal{D}, G)$ where \cup {*definitizer*(\mathcal{D}, G)}, *restrictors*(\mathcal{D}, G) $\triangleq \{ \mathbf{p}_i \leftarrow \mathbf{g}_i^i \mid 1 \le i \le n, 1 \le j \le m \},\$ definitizer(\mathcal{D}, \mathcal{G}) \triangleq g \leftarrow p₁, \cdots , p_n,

and where all atoms p_i and g, are distinct and fresh, and every occurrence of the atom g_i in \mathcal{P}_i , gives rise to an occurrence of the "tagged" atom g_i^i inside the disjoint union $\biguplus D(\mathcal{D})$.

Note: \mathcal{D} is finite, thus so is $D(\mathcal{D}) := \{\mathcal{P}_1, \ldots, \mathcal{P}_n\}$.

Encoding of DLP into LP

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Encoding of DLP into LP

Let's examine the LP game for g w.r.t. $\mathcal{P}:$

Opponent begins by doubting g. Player is forced to play the only rule whose head is gr

 $g \leftarrow p_1, \cdots, p_n$.

Opponent now has to select a p_i .

This corresponds to her choice of a definite instantiation of \mathcal{D} . Player next gets to decide which element $g_j \in G$ he wants to restrict to, and this is exactly what the restrictors are for: Player chooses the rule $\mathbf{p} \in \sigma^i$

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Truth value spaces

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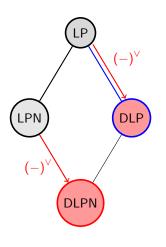
Summary & future research

Cool(?!) stuff

- abstract semantic framework, truth value spaces, ...
- semantic operator $(-)^{\vee}$
- "hand-made" DLP game
- \blacksquare infinite \implies first-order
- finite DLP \rightsquigarrow LP encoding

What's next?

- infinite DLP ~→ LP encoding;
- higher-order logic programming;
- coalgebraic semantics.



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Thanks!

Questions?

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Bonus tracks...

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Not is not not

Careful: \sim is not \neg

- $a \leftarrow \sim b$ $b \leftarrow \sim a$ $a \lor b \leftarrow$ all have different meanings.
- $\mathbf{a} \leftarrow \neg \mathbf{b} \quad \mathbf{b} \leftarrow \neg \mathbf{a} \quad \mathbf{a} \lor \mathbf{b} \leftarrow \text{ are equivalent in classical logic.}$