On the semantics of disjunctive logic programs

— Part II —

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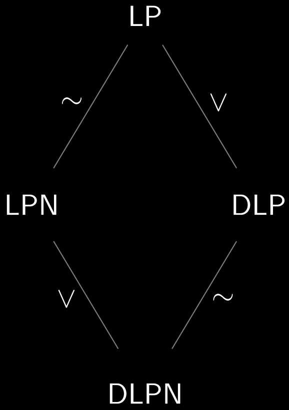
Previously on Semantics of Logic Programs...

Previously. . .



$\texttt{this} \leftarrow \texttt{that}$

Previously. . .

Example of LP

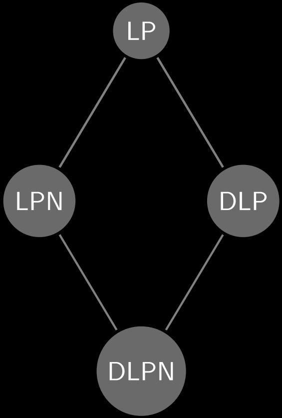
$$\left\{\begin{array}{r} \texttt{sleeps} \leftarrow \texttt{tired} \\ \texttt{works} \leftarrow \texttt{rested} \\ \texttt{eats} \leftarrow \texttt{rested}\ ,\ \texttt{hungry} \\ \texttt{rested} \leftarrow \end{array}\right\}$$

Queries

User:	$\leftarrow$ works	$\leftarrow$ tired	$\leftarrow$ eats
System:	Yes.	No.	No.

Semantics

Model-theoretic



Model-theoretic semantics

- **LP:** least Herbrand model
- **LPN:** well-founded model
- **DLP:** minimal models
- **DLPN:** ∞-valued minimal models

Previously. . .

The LP game

Example (1)

$$\mathcal{P} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{a}\ ,\ \mathtt{b} \\ \mathtt{p} \leftarrow \mathtt{c} \\ \mathtt{a} \leftarrow \mathtt{e} \\ \mathtt{b} \leftarrow \mathtt{d} \\ \mathtt{b} \leftarrow \mathtt{e} \\ \mathtt{e} \leftarrow \\ \mathtt{f} \leftarrow \end{array} \right\} \quad \left| \begin{array}{rl} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ \mathrm{B}_0: & \mathtt{p} \leftarrow \mathtt{a}\ ,\ \underline{\mathtt{b}} \\ \mathrm{B}_1: & \mathtt{b} \leftarrow \mathtt{d} \end{array} \right|$$

Believer lost! $\ddot\frown$

Previously. . .

The LP game

Example (2)

$$\mathcal{P} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{a}\ ,\ \mathtt{b} \\ \mathtt{p} \leftarrow \mathtt{c} \\ \mathtt{a} \leftarrow \mathtt{e} \\ \mathtt{b} \leftarrow \mathtt{d} \\ \mathtt{b} \leftarrow \mathtt{e} \\ \mathtt{e} \leftarrow \\ \mathtt{f} \leftarrow \end{array} \right\}$$

$$\left| \begin{array}{rl} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ B_0: & \mathtt{p} \leftarrow \underline{\mathtt{a}}\ ,\ \mathtt{b} \\ B_1: & \mathtt{a} \leftarrow \underline{\mathtt{e}} \\ B_2: & \mathtt{e} \leftarrow \emptyset \end{array} \right|$$

Believer wins! ⌣̈

Previously. . .

The LP game

Example (3)

$$\mathcal{Q} := \left\{ \begin{matrix} p \leftarrow q \\ q \leftarrow p \end{matrix} \right\} \qquad \left| \begin{array}{rr} \text{goal}: & \leftarrow \underline{p} \\ B_0: & p \leftarrow \underline{q} \\ B_1: & q \leftarrow \underline{p} \\ B_2: & p \leftarrow \underline{q} \\ B_3: & q \leftarrow \underline{p} \\ \vdots & \vdots \end{array} \right|$$

(benefit of the doubt) $\Longrightarrow$ **Believer lost!** ☹

Previously. . .

DLP game = LP game + combo + implicit

Example

$$\{\mathtt{a} \leftarrow \mathtt{a}, \quad \mathtt{b} \leftarrow \mathtt{b}, \quad \mathtt{c} \leftarrow \mathtt{c}, \quad \mathtt{p} \leftarrow \mathtt{p}\}$$

$$\mathcal{Q} := \left\{\begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{b} \leftarrow \mathtt{c} \\ \mathtt{a} \vee \mathtt{c} \leftarrow \quad \end{array}\right\} \quad \left|\begin{array}{lr} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ B_0: & \mathtt{p} \leftarrow \underline{\mathtt{a} \vee \mathtt{b}} \\ B_1: & \mathtt{a} \vee \mathtt{b} \leftarrow \underline{\mathtt{a} \vee \mathtt{c}} \\ B_2: & \mathtt{a} \vee \mathtt{c} \leftarrow \emptyset \end{array}\right|$$

Believer wins! $\ddot\smile$

Previously. . .

DLP game = LP game + combo + implicit

Example

$$\{\mathtt{a} \leftarrow \mathtt{a}, \quad \mathtt{b} \leftarrow \mathtt{b}, \quad \mathtt{c} \leftarrow \mathtt{c}, \quad \mathtt{p} \leftarrow \mathtt{p}\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{b} \leftarrow \mathtt{c} \\ \mathtt{a} \vee \mathtt{c} \leftarrow \phantom{\mathtt{c}} \end{array} \right\} \quad \left| \begin{array}{lr} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ B_0: & \mathtt{p} \leftarrow \underline{\mathtt{a} \vee \mathtt{b}} \\ B_1: & \mathtt{a} \vee \mathtt{b} \leftarrow \underline{\mathtt{a} \vee \mathtt{c}} \\ B_2: & \mathtt{a} \vee \mathtt{c} \leftarrow \emptyset \end{array} \right|$$

Believer wins! ⌣̈

Previously. . .

DLP game = LP game + combo + implicit

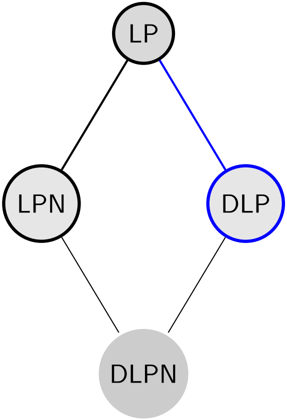
Example

$$\{a \leftarrow a, \quad b \leftarrow b, \quad c \leftarrow c, \quad p \leftarrow p\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\} \quad \left| \begin{array}{lr} \text{goal}: & \leftarrow \underline{p} \\ B_0: & p \leftarrow \underline{a \vee b} \\ B_1: & a \vee b \leftarrow \underline{a \vee c} \\ B_2: & a \vee c \leftarrow \emptyset \end{array} \right|$$

Believer wins! ⌣̈

Where we are



Model-theoretic semantics

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LPN: well-founded model

DLP: minimal models

DLPN: ∞-valued minimal models

Game semantics

LP: Di Cosmo, Loddo & Nicolet (1998)

LPN: Rondogiannis & Wadge (2005)

DLP: me (2013)

DLPN: ?

The DLP and LPN games

The DLP game

DLP game = LP game + combo + implicit

The LPN game

LPN game = LP game + rôle-switch

The DLP and LPN games

The DLP game

DLP game = LP game + combo + implicit

The LPN game

LPN game = LP game + rôle-switch

A game semantics for LPN

The LPN game

- Opponent vs Player
- Whenever a doubter doubts a negated atom $\sim p$, the rôles of the players switch: the believer becomes the doubter, doubting p.
- This implies draws!

A game semantics for LPN

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- Opponent vs Player
- Whenever a doubter doubts a negated atom $\sim\mathtt{p}$,
  the rôles of the players switch:
  the believer becomes the doubter, doubting $\mathtt{p}$.
- This implies draws!

The LPN game

Example plays (1)

$$\mathcal{P} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \\ \mathtt{q} \leftarrow \sim\mathtt{p} \\ \mathtt{r} \leftarrow \sim\mathtt{q} \end{array} \right\},$$

$$\pi_1 := \left| \begin{array}{lr} \text{goal}: & \leftarrow \underline{\mathtt{q}} \\ \mathsf{P}_0: & \mathtt{q} \leftarrow \underline{\sim\mathtt{p}} \\ \hline \mathsf{O}_2: & \mathtt{p} \leftarrow \phantom{\sim\mathtt{p}} \end{array} \right|, \qquad \pi_2 := \left| \begin{array}{lr} \text{goal}: & \leftarrow \underline{\mathtt{r}} \\ \mathsf{P}_0: & \mathtt{r} \leftarrow \underline{\sim\mathtt{q}} \\ \hline \mathsf{O}_2: & \mathtt{q} \leftarrow \underline{\sim\mathtt{p}} \\ \hline \mathsf{P}_3: & \mathtt{p} \leftarrow \phantom{\sim\mathtt{p}} \end{array} \right|.$$

The LPN game

Example plays (1)

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$$\pi_1 := \left| \begin{array}{rr} \text{goal}: & \leftarrow \underline{\mathtt{q}} \\ \mathsf{P}_0: & \mathtt{q} \leftarrow \underline{\underline{{\sim}\mathtt{p}}} \\ \hline \mathsf{O}_2: & \mathtt{p} \leftarrow \end{array} \right|, \qquad \pi_2 := \left| \begin{array}{rr} \text{goal}: & \leftarrow \underline{\mathtt{r}} \\ \mathsf{P}_0: & \mathtt{r} \leftarrow \underline{\underline{{\sim}\mathtt{q}}} \\ \hline \mathsf{O}_2: & \mathtt{q} \leftarrow \underline{\underline{{\sim}\mathtt{p}}} \\ \hline \mathsf{P}_3: & \mathtt{p} \leftarrow \end{array} \right|.$$

The LPN game

Example plays (2)

How about this infamous program of LPN:

$$\mathcal{Q} := \left\{ \begin{matrix} \mathtt{p} \leftarrow {\sim}\mathtt{q} \\ \mathtt{q} \leftarrow {\sim}\mathtt{p} \end{matrix} \right\} \qquad \pi_3 := \left| \begin{array}{rr} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ \mathsf{P}_0: & \mathtt{p} \leftarrow \underline{\underline{{\sim}\underline{\mathtt{q}}}} \\ \hline \mathsf{O}_2: & \mathtt{q} \leftarrow \underline{\underline{{\sim}\underline{\mathtt{p}}}} \\ \hline \mathsf{P}_3: & \mathtt{p} \leftarrow \underline{\underline{{\sim}\underline{\mathtt{q}}}} \\ \hline \mathsf{O}_5: & \mathtt{q} \leftarrow \underline{\underline{{\sim}\underline{\mathtt{p}}}} \\ \hline \vdots & \end{array} \right| .$$

Here no player is able to secure the believer rôle for themselves, so this play is won by neither of them: *the outcome is a tie*.

Truth values for NAF

Definition

$$\mathbb{B} : \mathbf{F} < \mathbf{T}$$

$$\mathbb{V} : \mathbf{F} < \mathbf{U} < \mathbf{T}$$

$$\mathbb{V}_\omega : \mathbf{F}_0 < \mathbf{F}_1 < \mathbf{F}_2 < \cdots < \mathbf{U} < \cdots < \mathbf{T}_2 < \mathbf{T}_1 < \mathbf{T}_0$$

$$\mathbb{V}_\kappa : \mathbf{F}_0 < \mathbf{F}_1 < \cdots < \mathbf{F}_\alpha < \cdots < \mathbf{U} < \cdots < \mathbf{T}_\alpha < \cdots < \mathbf{T}_1 < \mathbf{T}_0$$

Negation

$${\sim}\mathbf{F} = \mathbf{T} \qquad {\sim}\mathbf{U} = \mathbf{U} \qquad {\sim}\mathbf{T} = \mathbf{F}$$

$${\sim}\mathbf{F}_n = \mathbf{T}_{n+1} \qquad {\sim}\mathbf{T}_n = \mathbf{F}_{n+1}$$

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Payoff function

Definition (Payoff)

Let π be a play in some LPN game.
Then the **payoff** functions Φ_ω and Φ are defined by:

$$\Phi_\omega(\pi) \triangleq \begin{cases} \mathbf{T}_n, & \text{if Player wins in } \pi, \\ \mathbf{F}_n, & \text{if Player loses in } \pi, \\ \mathbf{U}, & \text{otherwise,} \end{cases}$$

where n is the number of rôle-switching moves played in π; and

$$\Phi \triangleq \mathit{collapse} \circ \Phi_\omega,$$

where *collapse* is the "subscript-removing" function.

Example payoffs

$$
\underbrace{\left|\begin{array}{rl}
\text{goal}: & \leftarrow \underline{\mathtt{q}} \\
P_0: & \mathtt{q} \leftarrow \underline{\underline{\sim\mathtt{p}}} \\ \hline
O_2: & \mathtt{p} \leftarrow
\end{array}\right|}_{\pi_1}
\qquad
\underbrace{\left|\begin{array}{rl}
\text{goal}: & \leftarrow \underline{\mathtt{r}} \\
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O_2: & \mathtt{q} \leftarrow \underline{\underline{\sim\mathtt{p}}} \\ \hline
P_3: & \mathtt{p} \leftarrow
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\qquad
\underbrace{\left|\begin{array}{rl}
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\vdots &
\end{array}\right|}_{\pi_3}
$$

The corresponding payoffs are:

$$\Phi_\omega(\pi_1) = \mathbf{F}_1, \qquad \Phi_\omega(\pi_2) = \mathbf{T}_2, \qquad \Phi_\omega(\pi_3) = \mathbf{U}.$$

$$\Phi(\pi_1) = \mathbf{F}, \qquad \Phi(\pi_2) = \mathbf{T}, \qquad \Phi(\pi_3) = \mathbf{U}.$$

Example payoffs

$$
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The corresponding payoffs are:

$$
\Phi_\omega(\pi_1) = \mathbf{F}_1, \qquad \Phi_\omega(\pi_2) = \mathbf{T}_2, \qquad \Phi_\omega(\pi_3) = \mathbf{U}.
$$

$$
\Phi(\pi_1) = \mathbf{F}, \qquad \Phi(\pi_2) = \mathbf{T}, \qquad \Phi(\pi_3) = \mathbf{U}.
$$

How do we get a semantics out of **LPNG**?

3-valued

$$\text{The value of q is}\ldots \begin{cases} \mathbf{T}, & \text{if there is a winning strategy} \\ \mathbf{U}, & \text{else, if there is a non-losing strategy} \\ \mathbf{F}, & \text{otherwise.} \end{cases}$$

∞-valued

The value of q is...

$$\sup \left\{ \inf \left\{ \Phi_\omega(\pi) \mid \pi \in \sigma \right\} \,\middle|\, \sigma \text{ is a strategy for } q \right\}.$$

How do we get a semantics out of **LPNG**?

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The value of q is... $\begin{cases} \mathbf{T}, & \text{if there is a winning strategy} \\ \mathbf{U}, & \text{else, if there is a non-losing strategy} \\ \mathbf{F}, & \text{otherwise.} \end{cases}$

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Truth value spaces

Defininition

A *truth value space* $\mathcal{V}$ is a completely distributive Heyting algebra with an additional unary operator $\sim$.

Weaponry

$$\top, \quad \bot, \quad x < y, \quad x > y$$

$$x \wedge y, \quad x \vee y, \quad \bigwedge S, \quad \bigvee S, \quad {\sim}x, \quad x \Rightarrow y$$

... and they all behave!

Truth value spaces

Examples of truth value spaces

$$\mathbb{B}, \quad \mathbb{V}, \quad \mathbb{V}_\omega, \quad \mathbb{V}_\kappa, \quad \ldots$$

The total order of the bounded set $\mathbb{V}_\kappa$ determines:

$$\begin{aligned} x \vee y &= \max\{x, y\}, \\ x \wedge y &= \min\{x, y\}, \end{aligned} \quad \text{and} \quad x \Rightarrow y = \begin{cases} \top & \text{if } x \leq y, \\ y & \text{otherwise.} \end{cases}$$

It remains to define the operator $\sim$:

$$\sim x \triangleq \begin{cases} \mathbf{T}_{\alpha+1} & \text{if } x = \mathbf{F}_\alpha, \\ \mathbf{F}_{\alpha+1} & \text{if } x = \mathbf{T}_\alpha, \\ \mathbf{U} & \text{if } x = \mathbf{U}. \end{cases}$$

An abstract semantic framework

ASF (1)

Definition

Let:

$$
\begin{aligned}
L &: \text{ a logic programming language,} \\
\mathcal{M} &: \text{ a set of “meanings”,} \\
\mathcal{V} &: \text{ a truth value space.}
\end{aligned}
$$

Then:

$\mathcal{M}$-semantics for L	$\mathbf{m} : \mathbf{P}_L \to \mathcal{M};$
$\mathcal{V}$-answer function for $\mathcal{M}$	$\mathbf{a} : \mathcal{M} \to \mathbf{Q}_L \to \mathcal{V};$
$\mathcal{V}$-system for L	$\mathbf{s} : \mathbf{P}_L \to \mathbf{Q}_L \to \mathcal{V};$
semantics for L	$(\mathbf{m}, \mathbf{a}) \rightsquigarrow \mathbf{a} \circ \mathbf{m} : \mathbf{P}_L \to \mathbf{Q}_L \to \mathcal{V}.$

An abstract semantic framework

ASF (2)

Defining the notions of...

- truth value space $\mathcal{V}$ (e.g., $\mathbb{B}$, $\mathbb{V}_\kappa$, ...)
- semantics of a language L as objects of study

$$\mathbf{s} \; : \; \mathbf{P}_L \times \mathbf{Q}_L \to \mathcal{V};$$

- equivalence of semantics ($\approx$);
- refinement of semantics ($\lhd$);
- ...

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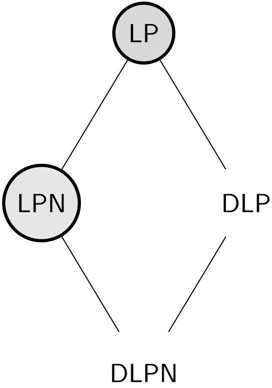
The semantic operator $(-)^\vee$



Transforming semantics

- Start with any semantics s for a non-disjunctive language.
- Apply the operator to get a new semantics $(s)^\vee$ for the corresponding disjunctive language.

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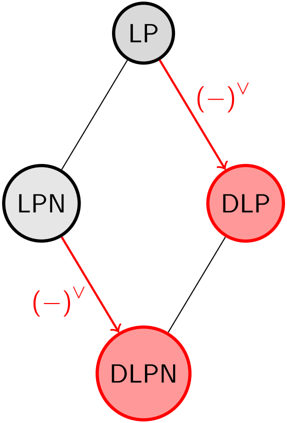

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Properties of $(-)^\vee$

Preservation properties

The operator respects equivalences and refinements:

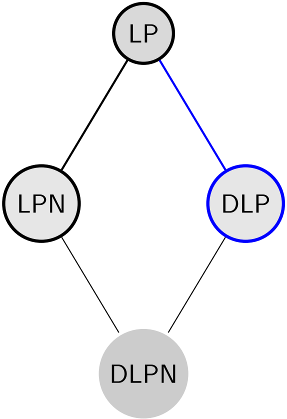
$$s_1 \approx s_2 \implies (s_1)^\vee \approx (s_2)^\vee$$
$$s_1 \lhd s_2 \implies (s_1)^\vee \lhd (s_2)^\vee$$

Expected outcomes

The operator yields the expected results for the standard semantics:

$$\text{(Least Herbrand Model)}^\vee \approx \text{Minimal Models}$$
$$\text{(Well-Founded Model)}^\vee \approx \infty\text{-valued Minimal Models}$$

Applications of $(-)^\vee$ on game semantics



Game semantics

LP: Di Cosmo, Loddo & Nicolet (1998)

LPN: Rondogiannis & Wadge (2005)

DLP: me (2013)

DLPN: ?

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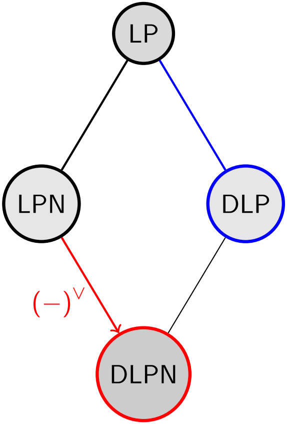

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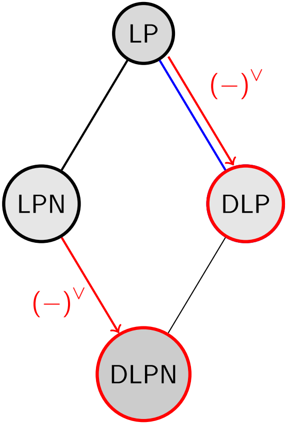

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Definite instantiations

Consider the disjunctive program

$$\mathcal{D} := \left\{ \begin{array}{l} \mathtt{s} \vee \mathtt{t} \leftarrow \mathtt{p}, \mathtt{b}, \mathtt{c} \\ \mathtt{a} \vee \mathtt{b} \leftarrow \\ \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b}, \mathtt{d}, \mathtt{f} \\ \mathtt{b} \vee \mathtt{c} \leftarrow \end{array} \right\}.$$

The set $\mathrm{D}(\mathcal{D})$ of its definite instantiations is

$$\mathrm{D}(\mathcal{D}) = \{\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \mathcal{P}_4, \mathcal{P}_5, \mathcal{P}_6, \mathcal{P}_7, \mathcal{P}_8\}.$$

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Definition of $(-)^{\vee}$ for a semantics **s** of LP

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\begin{array}{lrl}
\text{if} & \mathbf{s} & : \ \mathbf{P}_{\mathsf{LP}} \times \mathbf{Q}_{\mathsf{LP}} \to \mathcal{V}, \\
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Given. . .

an LP semantics **s**,
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What is $(\mathbf{LPG})^{\vee}$?

An example of $(-)^{\vee}$ in use

$$\mathcal{D} := \left\{ \begin{array}{l} \mathtt{s} \vee \mathtt{t} \leftarrow \mathtt{p}, \mathtt{b}, \mathtt{c} \\ \mathtt{a} \vee \mathtt{b} \leftarrow \\ \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b}, \mathtt{d}, \mathtt{f} \\ \mathtt{b} \vee \mathtt{c} \leftarrow \end{array} \right\}$$

goal : $\leftarrow \mathtt{p} \vee \mathtt{s} \vee \mathtt{t}$

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D_0 :

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What is $(\mathbf{LPG})^{\vee}$?

An example of $(-)^{\vee}$ in use

$$\mathcal{P}_4 := \left\{ \begin{array}{l} \texttt{t} \leftarrow \texttt{p} , \texttt{b} , \texttt{c} \\ \texttt{b} \leftarrow \\ \texttt{p} \leftarrow \texttt{a} \\ \texttt{p} \leftarrow \texttt{b} , \texttt{d} , \texttt{f} \\ \texttt{c} \leftarrow \end{array} \right\}$$

goal : $\leftarrow \texttt{p} \vee \texttt{s} \vee \texttt{t}$

D_0 : $\mathcal{P}_4$

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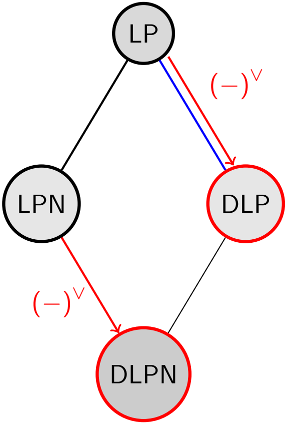
Applications of $(-)^\vee$



Model-theoretic semantics

- **LP:** least Herbrand model
- **LPN:** well-founded model
- **DLP:** minimal models
- **DLPN:** ∞-valued minimal models

Game semantics

- **LP:** Di Cosmo, Loddo & Nicolet (1998)
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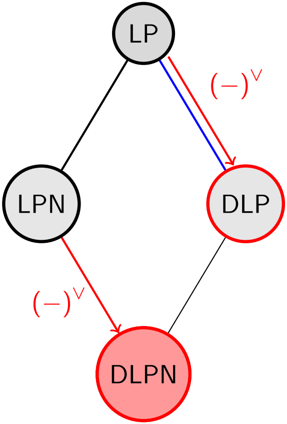

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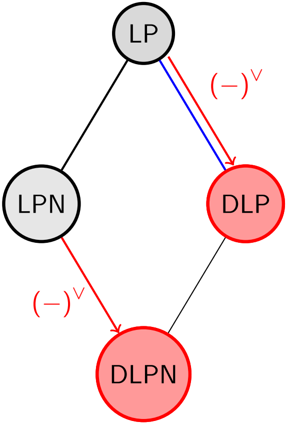

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Encoding of DLP into LP

The idea: suppose that we are given...

a **finite** DLP program $\mathcal{D}$ and a DLP goal $G \coloneqq \{g_1, \ldots, g_m\}$.

We can encode both DLP objects DLP into objects of LP with:

$$\mathit{encode} \ : \ \mathbf{P}_{\mathrm{DLP}} \times \mathbf{Q}_{\mathrm{DLP}} \to \mathbf{P}_{\mathrm{LP}} \times \mathbf{Q}_{\mathrm{LP}},$$

so that if $\mathit{encode}(\mathcal{D}, G) = (\mathcal{P}, g)$, we can use the LP game on $\mathcal{P}$ with the goal g to obtain an answer for the initial DLP goal G, w.r.t. the initial program $\mathcal{D}$.

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Encoding of DLP into LP

Given $\mathrm{D}(\mathcal{D}) \coloneqq \{\mathcal{P}_1, \ldots, \mathcal{P}_n\}$ and $G \coloneqq \mathrm{g}_1 \vee \cdots \vee \mathrm{g}_m$, define:

$$\mathit{encode}(\mathcal{D}, G) \triangleq (\mathcal{P}, g)$$

where

$$\mathcal{P} \coloneqq \mathcal{P}_1 \uplus \cdots \uplus \mathcal{P}_n \cup \mathit{restrictors}(\mathcal{D}, G) \cup \{\mathit{definitizer}(\mathcal{D}, G)\},$$

$$\mathit{restrictors}(\mathcal{D}, G) \triangleq \left\{\mathrm{p}_i \leftarrow \mathrm{g}_j^i \mid 1 \leq i \leq n,\ 1 \leq j \leq m\right\},$$

$$\mathit{definitizer}(\mathcal{D}, G) \triangleq \mathrm{g} \leftarrow \mathrm{p}_1\ ,\ \cdots\ ,\ \mathrm{p}_n,$$

and where all atoms p_i and g, are distinct and fresh, and every occurrence of the atom g_j in $\mathcal{P}_i$, gives rise to an occurrence of the "tagged" atom g_j^i inside the disjoint union $\biguplus \mathrm{D}(\mathcal{D})$.

Note: $\mathcal{D}$ is finite, thus so is $\mathrm{D}(\mathcal{D}) \coloneqq \{\mathcal{P}_1, \ldots, \mathcal{P}_n\}$.

Encoding of DLP into LP

Let's examine the LP game for g w.r.t. $\mathcal{P}$:

Opponent begins by doubting g.
Player is forced to play the only rule whose head is g:

$$\mathrm{g} \leftarrow \mathrm{p}_1\ ,\ \cdots\ ,\ \mathrm{p}_n.$$

Opponent now has to select a p_i.
This corresponds to her choice of a definite instantiation of $\mathcal{D}$.
Player next gets to decide which element $g_j \in G$ he wants to restrict to, and this is exactly what the restrictors are for:
Player chooses the rule $\mathrm{p}_i \leftarrow \mathrm{g}'_j$.
Opponent has no choice but to doubt the only conjunct in the body of that rule, g'_j, and finally we have arrived in this tagged atom, and so the game is successfully restricted within the rules of the correspondingly tagged $\mathcal{P}_j$.

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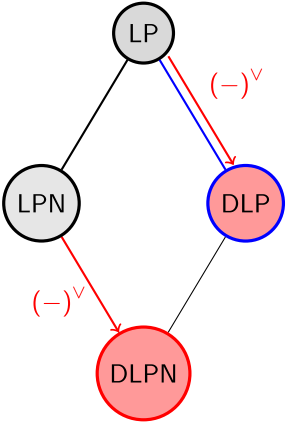
Summary & future research



Cool(?!) stuff

- abstract semantic framework, truth value spaces, ...
- semantic operator $(-)^{\vee}$
- "hand-made" DLP game
- infinite $\Longrightarrow$ first-order
- finite DLP $\rightsquigarrow$ LP encoding

What's next?

- infinite DLP $\rightsquigarrow$ LP encoding;
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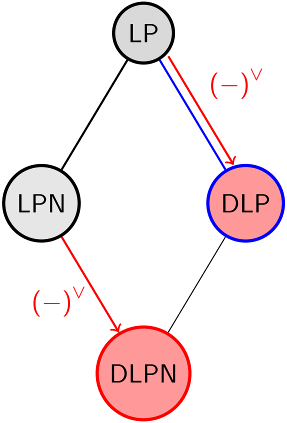

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Thanks!

Questions?

$\ominus$.

Bonus tracks. . .

Not is not not

Careful: $\sim$ is *not* $\neg$

$\texttt{a} \leftarrow \sim\texttt{b}$ $\quad \texttt{b} \leftarrow \sim\texttt{a}$ $\quad \texttt{a} \vee \texttt{b} \leftarrow$ all have different meanings.

$\texttt{a} \leftarrow \neg\texttt{b}$ $\quad \texttt{b} \leftarrow \neg\texttt{a}$ $\quad \texttt{a} \vee \texttt{b} \leftarrow$ are equivalent in classical logic.