On the semantics of disjunctive logic programs

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PhD thesis defense
July 2nd, 2014

Programming languages: syntax and semantics

Syntax

How programs are written.

Semantics

What programs mean.

- Denotational semantics
- Operational semantics
- Axiomatic semantics
- ...

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Programming languages: paradigms

Imperative

Describe *how* to solve the problem.
Describe *how* the program computes the solution.

vs.

Declarative

Describe *what* the problem is.
Describe *what* is a solution.

Programming languages: paradigms

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Describe *how* to solve the problem.
Describe *how* the program computes the solution.

vs.

Declarative: Logic programming

Describe *what* the problem is.
Describe *what* is a solution.

Four languages: the big picture

$$\begin{tikzcd}[column sep=large, row sep=huge]
 & \mathrm{LP} \arrow[dl, dash, "\sim"'] \arrow[dr, dash, "\vee"] & \\
\mathrm{LPN} \arrow[dr, dash, "\vee"'] & & \mathrm{DLP} \arrow[dl, dash, "\sim"] \\
 & \mathrm{DLPN} &
\end{tikzcd}$$

Examples of logic programs

Example of LP

$$\left\{\begin{aligned} \mathtt{sleeps} &\leftarrow \mathtt{tired} \\ \mathtt{works} &\leftarrow \mathtt{rested} \\ \mathtt{eats} &\leftarrow \mathtt{rested}\ ,\ \mathtt{hungry} \\ \mathtt{rested} &\leftarrow \end{aligned}\right\}$$

Queries

User:	$\leftarrow$ works	$\leftarrow$ eats
System:	Yes.	No.

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Examples of logic programs

Example of LPN

$$\left\{\begin{array}{r} \texttt{sleeps} \leftarrow \texttt{tired} \\ \texttt{works} \leftarrow {\sim}\texttt{tired} \\ \texttt{eats} \leftarrow {\sim}\texttt{tired}\ ,\ \texttt{hungry} \end{array}\right\}$$

Queries

U:	$\leftarrow$ sleeps	$\leftarrow$ works
S:	No.	Yes.

Note: $\sim$ is *not* $\neg$.
$\{\texttt{a} \leftarrow {\sim}\texttt{b}\}$, $\{\texttt{b} \leftarrow {\sim}\texttt{a}\}$, and $\{\texttt{a} \vee \texttt{b} \leftarrow\}$ have different meanings.

Examples of logic programs

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Examples of logic programs

Example of DLP

$$\left\{\begin{array}{r}\texttt{mathematician} \leftarrow \texttt{topologist}\\ \texttt{mathematician} \leftarrow \texttt{algebraist}\\ \texttt{algebraist} \vee \texttt{topologist} \leftarrow \qquad\qquad\quad\end{array}\right\}$$

Queries

U:	$\leftarrow \texttt{algebraist}$	$\leftarrow \texttt{topologist}$	$\leftarrow \texttt{mathematician}$
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Examples of logic programs

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Queries

U:	$\leftarrow$ `algebraist`	$\leftarrow$ `topologist`	$\leftarrow$ `mathematician`
S:	`No.`	`No.`	`Yes.`

First-order logic programs

How first-order programs look like

$$
\left\{
\begin{array}{rl}
\texttt{father}(X, Y) \leftarrow & \texttt{male}(X), \texttt{parent}(X, Y) \\
\texttt{mother}(X, Y) \leftarrow & \texttt{female}(X), \texttt{parent}(X, Y) \\
\texttt{grandparent}(X, Z) \leftarrow & \texttt{parent}(X, Y), \texttt{parent}(Y, Z) \\
\texttt{male}(\mathit{stan}) \leftarrow & \\
\texttt{female}(\mathit{sharon}) \leftarrow & \\
\texttt{parent}(\mathit{randy}, \mathit{stan}) \leftarrow & \\
\texttt{parent}(\mathit{marvin}, \mathit{randy}) \leftarrow & \\
\vdots &
\end{array}
\right\}
$$

First-order logic programs

Finite propositional programs. . .

. . . do *not* suffice to provide semantics to first-order programs; but **infinite** propositional programs, do.

Syntax of logic programs

What is a logic program?

It is *a set of rules*:

$$\mathtt{a} \leftarrow \mathtt{b}_1 \;,\; \ldots \;,\; \mathtt{b}_n \qquad \text{(LP)}$$

$$\mathtt{a}_1 \vee \cdots \vee \mathtt{a}_k \leftarrow \mathtt{b}_1 \;,\; \ldots \;,\; \mathtt{b}_n \qquad \text{(DLP)}$$

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Definition (Four programming languages)

A **DLPN program** is a set of DLPN-rules.

A rule without a head (**goal**) represents a **query** to the system:

$$\leftarrow \mathtt{p} \qquad \text{LP(N)}$$

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Semantics of logic programs

What to expect

- Assigning truth values to whatever needs a truth value.
- Deciding which goals fail and which succeed (and how much).

Lack of information

If we have no reason to believe something, we don't.

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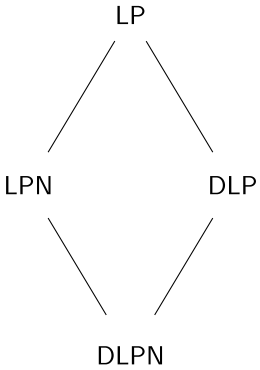
Models in the big picture



Model-theoretic semantics

LP: least Herbrand model
(van Emden & Kowalski, 1976)

LPN: well-founded model
(Van Gelder et al., 1991)
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DLP: minimal models
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DLPN: ∞-valued minimal models
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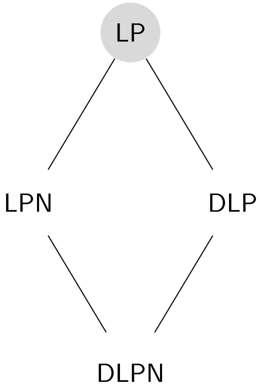

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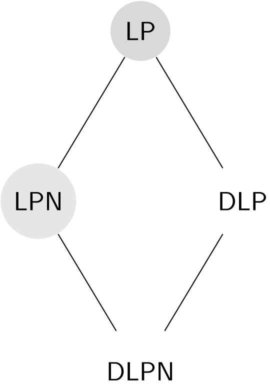

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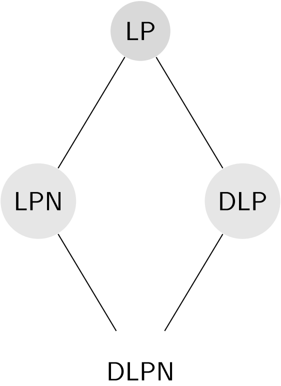

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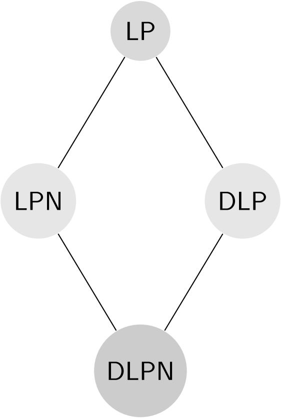

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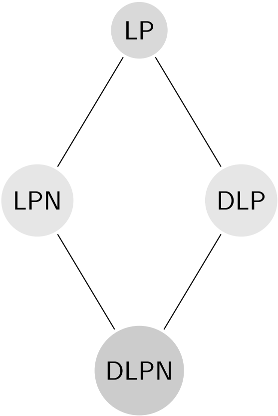

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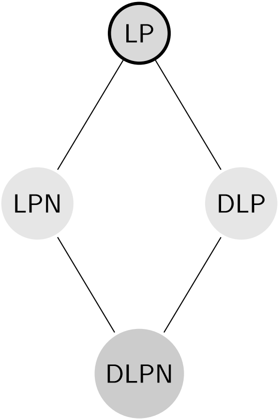

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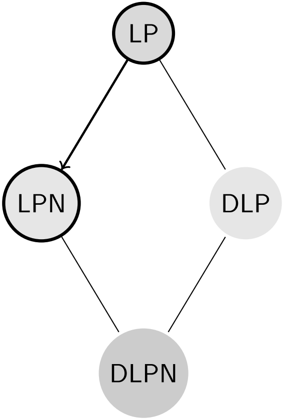

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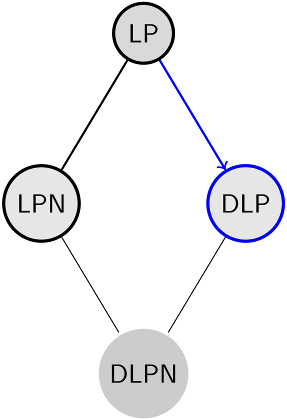

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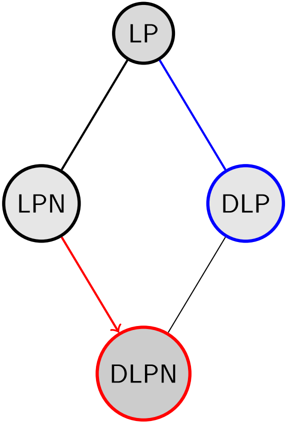

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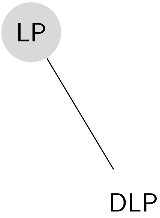

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Herbrand base, interpretations, and models through an example

$$\mathcal{P} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{a}\ ,\ \mathtt{b} \\ \mathtt{p} \leftarrow \mathtt{c} \\ \mathtt{a} \leftarrow \mathtt{e} \\ \mathtt{b} \leftarrow \mathtt{d} \\ \mathtt{b} \leftarrow \mathtt{e} \\ \mathtt{e} \leftarrow \\ \mathtt{f} \leftarrow \end{array} \right\}$$

- At least one Herbrand model exists (the Herbrand Base)
- Model intersection property (mip)

$\therefore$ Existence of a $\subseteq$-least Herbrand model (LHM).
This must be the model the programmer had in mind:
it provides the semantics.

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Herbrand base $\{\mathtt{p}, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}, \mathtt{e}, \mathtt{f}\}$

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Herbrand interpretations	$\{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$
	$\{\mathtt{e}, \mathtt{f}\}$
	$\emptyset$
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LP

Herbrand base, interpretations, and models through an example

$$\mathcal{P} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{a}\ ,\ \mathtt{b} \\ \mathtt{p} \leftarrow \mathtt{c} \\ \mathtt{a} \leftarrow \mathtt{e} \\ \mathtt{b} \leftarrow \mathtt{d} \\ \mathtt{b} \leftarrow \mathtt{e} \\ \mathtt{e} \leftarrow \emptyset \\ \mathtt{f} \leftarrow \emptyset \end{array} \right\}$$

Herbrand base $\{\mathtt{p}, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}, \mathtt{e}, \mathtt{f}\}$

Herbrand interpretations

$\{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$

$\{\mathtt{e}, \mathtt{f}\}$

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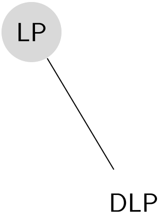
DLP

Model-theoretic semantics



DLP

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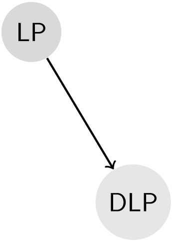

DLP

An example

Example

Consider the disjunctive program

$$\mathcal{P} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{a} \vee \mathtt{b} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}.$$

It has three models:

$$\{\mathtt{a},\mathtt{p}\}, \quad \{\mathtt{b},\mathtt{p}\}, \quad \{\mathtt{a},\mathtt{b},\mathtt{p}\},$$

none of which is least! (However, the first two are *minimal*.)
Notice that the intersection of all its models is $\{\mathtt{p}\}$, which is *not* a model.

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DLP

Model-theoretic semantics

Difficulties

- We no longer have a least model. $\ddot\frown$
- We have a set of minimal models—but there's no mip!

Approach (Minker 1982)

- We can consider this set to be the meaning of our program.
- The goal $\leftarrow \mathtt{G}$ succeeds if $\mathtt{G}$ is $\mathbf{T}$ in every minimal model of $\mathcal{P}$.

DLP

Model-theoretic semantics

Example

Consider the DLP program

$$\mathcal{Q} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{b} \leftarrow \mathtt{c} \\ \mathtt{a} \vee \mathtt{c} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}.$$

Look at its models: $\{a, p\}$, $\{a, b, p\}$, $\{c, b, p\}$, $\{a, b, c, p\}$.
E.g., $a \vee b$ is **T**, while a is **F**.

DLP

Model-theoretic semantics

Example

Consider the DLP program

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DLP

Model-theoretic semantics

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Consider the DLP program

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}.$$

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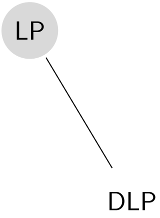
Game semantics

Why games?

- Nice denotational semantics for LP.
- We want to add more features in our language ($\sim$ and $\vee$).
- It becomes difficult to deal with them in a uniform way.
- Instead we look at **games**.

LP

LP game semantics



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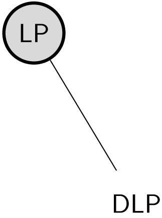

The LP game

The idea

Given a program $\mathcal{P}$ and a goal clause $\leftarrow$ q, a game $\Gamma_{\mathcal{P}}(\leftarrow \mathrm{q})$ will determine the goal's success and therefore q's truth value.

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Layout of the games...

- Two players (Doubter vs Believer, Opponent vs Player):
  - Doubter who doubts "things" from bodies of rules;
  - Believer who justifies "things" by playing rules.
- A player who can't make a legal move, loses.
- Doubter has the "*benefit of the doubt*".

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The LP game

Example plays (1)

$$\mathcal{P} := \left\{ \begin{array}{l} \texttt{p} \leftarrow \texttt{a} \;,\; \texttt{b} \\ \texttt{p} \leftarrow \texttt{c} \\ \texttt{a} \leftarrow \texttt{e} \\ \texttt{b} \leftarrow \texttt{d} \\ \texttt{b} \leftarrow \texttt{e} \\ \texttt{e} \leftarrow \\ \texttt{f} \leftarrow \end{array} \right\}$$

goal : $\leftarrow \texttt{p}$

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Believer lost! $\ddot{\frown}$

The LP game

Example plays (2)

$$\mathcal{P} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{a}\ ,\ \mathtt{b} \\ \mathtt{p} \leftarrow \mathtt{c} \\ \mathtt{a} \leftarrow \mathtt{e} \\ \mathtt{b} \leftarrow \mathtt{d} \\ \mathtt{b} \leftarrow \mathtt{e} \\ \mathtt{e} \leftarrow \\ \mathtt{f} \leftarrow \end{array} \right\}$$

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The LP game

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The LP game

Example plays (2)

$$\mathcal{P} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{a}\ ,\ \mathtt{b} \\ \mathtt{p} \leftarrow \mathtt{c} \\ \mathtt{a} \leftarrow \mathtt{e} \\ \mathtt{b} \leftarrow \mathtt{d} \\ \mathtt{b} \leftarrow \mathtt{e} \\ \mathtt{e} \leftarrow \\ \mathtt{f} \leftarrow \end{array} \right\}$$

$$\text{goal}: \quad \leftarrow \underline{\mathtt{p}}$$

The LP game

Example plays (2)

$$\mathcal{P} := \left\{ \begin{array}{l} \texttt{p} \leftarrow \texttt{a} \texttt{,} \texttt{b} \\ \texttt{p} \leftarrow \texttt{c} \\ \texttt{a} \leftarrow \texttt{e} \\ \texttt{b} \leftarrow \texttt{d} \\ \texttt{b} \leftarrow \texttt{e} \\ \texttt{e} \leftarrow \\ \texttt{f} \leftarrow \end{array} \right\}$$

goal :	$\leftarrow \underline{\texttt{p}}$
B_0 :	

The LP game

Example plays (2)

$$\mathcal{P} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{a}\ ,\ \mathtt{b} \\ \mathtt{p} \leftarrow \mathtt{c} \\ \mathtt{a} \leftarrow \mathtt{e} \\ \mathtt{b} \leftarrow \mathtt{d} \\ \mathtt{b} \leftarrow \mathtt{e} \\ \mathtt{e} \leftarrow \\ \mathtt{f} \leftarrow \end{array} \right\}$$

$$\left| \begin{array}{rl} \mathsf{goal}: & \leftarrow \underline{\mathtt{p}} \\ \mathsf{B}_0: & \mathtt{p} \leftarrow \mathtt{a}\ ,\ \mathtt{b} \end{array} \right|$$

The LP game

Example plays (2)

$$\mathcal{P} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{a}\ ,\ \mathtt{b} \\ \mathtt{p} \leftarrow \mathtt{c} \\ \mathtt{a} \leftarrow \mathtt{e} \\ \mathtt{b} \leftarrow \mathtt{d} \\ \mathtt{b} \leftarrow \mathtt{e} \\ \mathtt{e} \leftarrow \\ \mathtt{f} \leftarrow \end{array} \right\}$$

$$\left| \begin{array}{rr} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ B_0: & \mathtt{p} \leftarrow \mathtt{a}\ ,\ \mathtt{b} \end{array} \right|$$

The LP game

Example plays (2)

$$\mathcal{P} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{a}\ ,\ \mathtt{b} \\ \mathtt{p} \leftarrow \mathtt{c} \\ \mathtt{a} \leftarrow \mathtt{e} \\ \mathtt{b} \leftarrow \mathtt{d} \\ \mathtt{b} \leftarrow \mathtt{e} \\ \mathtt{e} \leftarrow \\ \mathtt{f} \leftarrow \end{array} \right\}$$

$$\left| \begin{array}{rl} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ B_0: & \mathtt{p} \leftarrow \underline{\mathtt{a}}\ ,\ \mathtt{b} \end{array} \right|$$

The LP game

Example plays (2)

$$\mathcal{P} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{a}\ ,\ \mathtt{b} \\ \mathtt{p} \leftarrow \mathtt{c} \\ \color{blue}{\mathtt{a} \leftarrow \mathtt{e}} \\ \mathtt{b} \leftarrow \mathtt{d} \\ \mathtt{b} \leftarrow \mathtt{e} \\ \mathtt{e} \leftarrow \\ \mathtt{f} \leftarrow \end{array} \right\}$$

$$\left| \begin{array}{rl} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ B_0: & \mathtt{p} \leftarrow \underline{\mathtt{a}}\ ,\ \mathtt{b} \\ B_1: & \end{array} \right|$$

The LP game

Example plays (2)

$$\mathcal{P} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{a}\ ,\ \mathtt{b} \\ \mathtt{p} \leftarrow \mathtt{c} \\ \mathtt{a} \leftarrow \mathtt{e} \\ \mathtt{b} \leftarrow \mathtt{d} \\ \mathtt{b} \leftarrow \mathtt{e} \\ \mathtt{e} \leftarrow \\ \mathtt{f} \leftarrow \end{array} \right\}$$

$$\left| \begin{array}{rl} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ \mathsf{B}_0: & \mathtt{p} \leftarrow \underline{\mathtt{a}}\ ,\ \mathtt{b} \\ \mathsf{B}_1: & \mathtt{a} \leftarrow \mathtt{e} \end{array} \right|$$

The LP game

Example plays (2)

$$\mathcal{P} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{a}\ ,\ \mathtt{b} \\ \mathtt{p} \leftarrow \mathtt{c} \\ \mathtt{a} \leftarrow \mathtt{e} \\ \mathtt{b} \leftarrow \mathtt{d} \\ \mathtt{b} \leftarrow \mathtt{e} \\ \mathtt{e} \leftarrow \\ \mathtt{f} \leftarrow \end{array} \right\}$$

$$\left| \begin{array}{rl} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ B_0: & \mathtt{p} \leftarrow \underline{\mathtt{a}}\ ,\ \mathtt{b} \\ B_1: & \mathtt{a} \leftarrow \color{red}{\mathtt{e}} \end{array} \right|$$

The LP game

Example plays (2)

$$\mathcal{P} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{a}\ ,\ \mathtt{b} \\ \mathtt{p} \leftarrow \mathtt{c} \\ \mathtt{a} \leftarrow \mathtt{e} \\ \mathtt{b} \leftarrow \mathtt{d} \\ \mathtt{b} \leftarrow \mathtt{e} \\ \mathtt{e} \leftarrow \\ \mathtt{f} \leftarrow \end{array} \right\}$$

$$\left| \begin{array}{rl} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ \mathsf{B}_0: & \mathtt{p} \leftarrow \underline{\mathtt{a}}\ ,\ \mathtt{b} \\ \mathsf{B}_1: & \mathtt{a} \leftarrow \underline{\mathtt{e}} \\ & \end{array} \right|$$

The LP game

Example plays (2)

$$\mathcal{P} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{a}\ ,\ \mathtt{b} \\ \mathtt{p} \leftarrow \mathtt{c} \\ \mathtt{a} \leftarrow \mathtt{e} \\ \mathtt{b} \leftarrow \mathtt{d} \\ \mathtt{b} \leftarrow \mathtt{e} \\ \color{blue}{\mathtt{e} \leftarrow} \\ \mathtt{f} \leftarrow \end{array} \right\}$$

$$\left| \begin{array}{rl} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ B_0: & \mathtt{p} \leftarrow \underline{\mathtt{a}}\ ,\ \mathtt{b} \\ B_1: & \mathtt{a} \leftarrow \underline{\mathtt{e}} \\ B_2: & \end{array} \right|$$

The LP game

Example plays (2)

$$\mathcal{P} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{a}\ ,\ \mathtt{b} \\ \mathtt{p} \leftarrow \mathtt{c} \\ \mathtt{a} \leftarrow \mathtt{e} \\ \mathtt{b} \leftarrow \mathtt{d} \\ \mathtt{b} \leftarrow \mathtt{e} \\ \mathtt{e} \leftarrow \\ \mathtt{f} \leftarrow \end{array} \right\}$$

$$\left| \begin{array}{rl} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ \mathsf{B}_0: & \mathtt{p} \leftarrow \underline{\mathtt{a}}\ ,\ \mathtt{b} \\ \mathsf{B}_1: & \mathtt{a} \leftarrow \underline{\mathtt{e}} \\ \mathsf{B}_2: & \mathtt{e} \leftarrow \end{array} \right|$$

The LP game

Example plays (2)

$$\mathcal{P} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{a}\ ,\ \mathtt{b} \\ \mathtt{p} \leftarrow \mathtt{c} \\ \mathtt{a} \leftarrow \mathtt{e} \\ \mathtt{b} \leftarrow \mathtt{d} \\ \mathtt{b} \leftarrow \mathtt{e} \\ \mathtt{e} \leftarrow \\ \mathtt{f} \leftarrow \end{array} \right\}$$

$$\left| \begin{array}{rl} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ B_0: & \mathtt{p} \leftarrow \underline{\mathtt{a}}\ ,\ \mathtt{b} \\ B_1: & \mathtt{a} \leftarrow \underline{\mathtt{e}} \\ B_2: & \mathtt{e} \leftarrow \emptyset \end{array} \right|$$

The LP game

Example plays (2)

$$\mathcal{P} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{a}\ ,\ \mathtt{b} \\ \mathtt{p} \leftarrow \mathtt{c} \\ \mathtt{a} \leftarrow \mathtt{e} \\ \mathtt{b} \leftarrow \mathtt{d} \\ \mathtt{b} \leftarrow \mathtt{e} \\ \mathtt{e} \leftarrow \\ \mathtt{f} \leftarrow \end{array} \right\}
\qquad
\left| \begin{array}{rl} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ \mathsf{B}_0: & \mathtt{p} \leftarrow \underline{\mathtt{a}}\ ,\ \mathtt{b} \\ \mathsf{B}_1: & \mathtt{a} \leftarrow \underline{\mathtt{e}} \\ \mathsf{B}_2: & \mathtt{e} \leftarrow \end{array} \right|$$

Believer wins! $\ddot\smile$

The LP game

Example plays (3)

$$\mathcal{Q} := \left\{ \begin{matrix} \mathtt{p} \leftarrow \mathtt{q} \\ \mathtt{q} \leftarrow \mathtt{p} \end{matrix} \right\}$$

goal :	$\leftarrow \mathtt{p}$

The LP game

Example plays (3)

$$\mathcal{Q} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{q} \\ \mathtt{q} \leftarrow \mathtt{p} \end{array} \right\}$$

goal :	$\leftarrow \mathtt{p}$

The LP game

Example plays (3)

$$\mathcal{Q} := \left\{ \begin{matrix} \mathtt{p} \leftarrow \mathtt{q} \\ \mathtt{q} \leftarrow \mathtt{p} \end{matrix} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$

The LP game

Example plays (3)

$$\mathcal{Q} := \left\{ \begin{matrix} \color{blue}{\mathtt{p} \leftarrow \mathtt{q}} \\ \mathtt{q} \leftarrow \mathtt{p} \end{matrix} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	

The LP game

Example plays (3)

$$\mathcal{Q} := \left\{ \begin{matrix} \mathtt{p} \leftarrow \mathtt{q} \\ \mathtt{q} \leftarrow \mathtt{p} \end{matrix} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	$\mathtt{p} \leftarrow \mathtt{q}$

The LP game

Example plays (3)

$$\mathcal{Q} := \left\{ \begin{matrix} \mathrm{p} \leftarrow \mathrm{q} \\ \mathrm{q} \leftarrow \mathrm{p} \end{matrix} \right\}$$

$$\left| \begin{array}{lr} \text{goal :} & \leftarrow \underline{\mathrm{p}} \\ \mathrm{B}_0 \text{ :} & \mathrm{p} \leftarrow \color{red}{\mathrm{q}} \end{array} \right|$$

The LP game

Example plays (3)

$$\mathcal{Q} := \left\{ \begin{matrix} \mathtt{p} \leftarrow \mathtt{q} \\ \mathtt{q} \leftarrow \mathtt{p} \end{matrix} \right\}$$

$$\left| \begin{array}{rr} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ \mathsf{B}_0: & \mathtt{p} \leftarrow \underline{\mathtt{q}} \end{array} \right|$$

The LP game

Example plays (3)

$$\mathcal{Q} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{q} \\ \mathtt{q} \leftarrow \mathtt{p} \end{array} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	$\mathtt{p} \leftarrow \underline{\mathtt{q}}$
B_1 :	

The LP game

Example plays (3)

$$\mathcal{Q} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{q} \\ \mathtt{q} \leftarrow \mathtt{p} \end{array} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	$\mathtt{p} \leftarrow \underline{\mathtt{q}}$
B_1 :	$\mathtt{q} \leftarrow \mathtt{p}$

The LP game

Example plays (3)

$$\mathcal{Q} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{q} \\ \mathtt{q} \leftarrow \mathtt{p} \end{array} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	$\mathtt{p} \leftarrow \underline{\mathtt{q}}$
B_1 :	$\mathtt{q} \leftarrow \mathtt{p}$

The LP game

Example plays (3)

$$\mathcal{Q} := \left\{ \begin{matrix} \mathtt{p} \leftarrow \mathtt{q} \\ \mathtt{q} \leftarrow \mathtt{p} \end{matrix} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	$\mathtt{p} \leftarrow \underline{\mathtt{q}}$
B_1 :	$\mathtt{q} \leftarrow \underline{\mathtt{p}}$

The LP game

Example plays (3)

$$\mathcal{Q} := \left\{ \begin{array}{l} \color{blue}{\mathtt{p} \leftarrow \mathtt{q}} \\ \mathtt{q} \leftarrow \mathtt{p} \end{array} \right\}$$

$$\begin{array}{|r r|} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ \mathsf{B}_0: & \mathtt{p} \leftarrow \underline{\mathtt{q}} \\ \mathsf{B}_1: & \mathtt{q} \leftarrow \underline{\mathtt{p}} \\ \mathsf{B}_2: & \end{array}$$

The LP game

Example plays (3)

$$\mathcal{Q} := \left\{ \begin{array}{c} \mathtt{p} \leftarrow \mathtt{q} \\ \mathtt{q} \leftarrow \mathtt{p} \end{array} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	$\mathtt{p} \leftarrow \underline{\mathtt{q}}$
B_1 :	$\mathtt{q} \leftarrow \underline{\mathtt{p}}$
B_2 :	$\mathtt{p} \leftarrow \mathtt{q}$

The LP game

Example plays (3)

$$\mathcal{Q} := \left\{ \begin{matrix} \mathtt{p} \leftarrow \mathtt{q} \\ \mathtt{q} \leftarrow \mathtt{p} \end{matrix} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	$\mathtt{p} \leftarrow \underline{\mathtt{q}}$
B_1 :	$\mathtt{q} \leftarrow \underline{\mathtt{p}}$
B_2 :	$\mathtt{p} \leftarrow \underline{\mathtt{q}}$

The LP game

Example plays (3)

$$\mathcal{Q} := \left\{ \begin{matrix} \mathtt{p} \leftarrow \mathtt{q} \\ \mathtt{q} \leftarrow \mathtt{p} \end{matrix} \right\}$$

$$\left| \begin{array}{rr} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ \mathsf{B}_0: & \mathtt{p} \leftarrow \underline{\mathtt{q}} \\ \mathsf{B}_1: & \mathtt{q} \leftarrow \underline{\mathtt{p}} \\ \mathsf{B}_2: & \mathtt{p} \leftarrow \underline{\mathtt{q}} \\ \mathsf{B}_3: & \mathtt{q} \leftarrow \underline{\mathtt{p}} \\ \mathsf{B}_4: & \mathtt{p} \leftarrow \underline{\mathtt{q}} \\ \vdots & \vdots \end{array} \right|$$

The LP game

Example plays (3)

$$\mathcal{Q} := \left\{ \begin{matrix} \mathtt{p} \leftarrow \mathtt{q} \\ \mathtt{q} \leftarrow \mathtt{p} \end{matrix} \right\} \qquad \left| \begin{array}{rr} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ \mathsf{B}_0: & \mathtt{p} \leftarrow \underline{\mathtt{q}} \\ \mathsf{B}_1: & \mathtt{q} \leftarrow \underline{\mathtt{p}} \\ \mathsf{B}_2: & \mathtt{p} \leftarrow \underline{\mathtt{q}} \\ \mathsf{B}_3: & \mathtt{q} \leftarrow \underline{\mathtt{p}} \\ \mathsf{B}_4: & \mathtt{p} \leftarrow \underline{\mathtt{q}} \\ \vdots & \vdots \end{array} \right|$$

(benefit of the doubt) $\implies$ **Believer lost!** $\ddot{\frown}$

The LP game

How to get semantics out of it

We look at strategies

The value of p w.r.t. $\mathcal{P}$ is determined by the game:
$\leftarrow$ p succeeds iff there is a winning strategy (for the Believer).

The connecting result

Theorem (Di Cosmo, Loddo & Nicolet)

LPG $\approx$ LHM

The LP game

How to get semantics out of it

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The value of p w.r.t. $\mathcal{P}$ is determined by the game:
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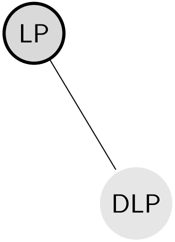
Theorem (Di Cosmo, Loddo & Nicolet)

$$\mathbf{LPG} \approx \mathbf{LHM}$$

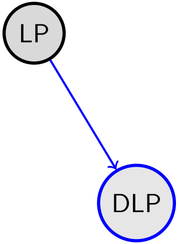
DLP

DLP game semantics



DLP

DLP game semantics



From LP to DLP (1)

Let's try the LP game

$$\mathcal{R} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \\ \mathtt{q} \leftarrow \mathtt{a} \end{array} \right\} \quad \Big| \, \text{goal :} \qquad \leftarrow \mathtt{p} \vee \mathtt{q} \qquad\qquad \Big|$$

From LP to DLP (1)

Let's try the LP game

$$\mathcal{R} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \\ \mathtt{q} \leftarrow \mathtt{a} \end{array} \right\} \quad \Big| \text{goal} : \qquad \leftarrow \mathtt{p} \vee \mathtt{q} \qquad \Big|$$

From LP to DLP (1)

Let's try the LP game

$$\mathcal{R} := \left\{ \begin{matrix} \mathtt{p} \leftarrow \\ \mathtt{q} \leftarrow \mathtt{a} \end{matrix} \right\} \quad \Big| \text{goal :} \qquad \leftarrow \underline{\mathtt{p} \vee \mathtt{q}} \qquad \Big|$$

From LP to DLP (1)

Let's try the LP game

$$\mathcal{R} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \\ \mathtt{q} \leftarrow \mathtt{a} \end{array} \right\} \quad \Big| \, \text{goal :} \qquad \leftarrow \underline{\mathtt{p} \vee \mathtt{q}} \qquad \Big|$$

Believer lost! ☹

From LP to DLP (1)

(1) Believer justifies subsets

Believer can justify any non-empty subset of the doubted disjunction.

From LP to DLP (1)

LP game + subset

$$\mathcal{R} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \\ \mathtt{q} \leftarrow \mathtt{a} \end{array} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p} \vee \mathtt{q}}$

From LP to DLP (1)

LP game + subset

$$\mathcal{R} := \left\{ \begin{array}{l} p \leftarrow \\ q \leftarrow a \end{array} \right\} \quad \left| \begin{array}{ll} \text{goal} : & \leftarrow \underline{p \vee q} \\ B_0 : & \end{array} \right|$$

From LP to DLP (1)

LP game + subset

$$\mathcal{R} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \\ \mathtt{q} \leftarrow \mathtt{a} \end{array} \right\} \quad \left| \begin{array}{rl} \text{goal}: & \quad \leftarrow \underline{\mathtt{p} \vee \mathtt{q}} \\ \mathsf{B}_0: & \mathtt{p} \leftarrow \end{array} \right|$$

From LP to DLP (1)

LP game + subset

$$\mathcal{R} := \left\{ \begin{array}{l} p \leftarrow \\ q \leftarrow a \end{array} \right\}$$

goal :		$\leftarrow \underline{p \vee q}$
B_0 :	p	$\leftarrow \emptyset$

Believer wins! $\ddot\smile$

From LP to DLP (2)

LP game + subset

$$\mathcal{P} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{a} \vee \mathtt{b} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}$$

goal : $\leftarrow \mathtt{p}$

From LP to DLP (2)

LP game + subset

$$\mathcal{P} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{a} \vee \mathtt{b} \leftarrow \phantom{\mathtt{a}} \end{array} \right\} \quad \left| \quad \text{goal :} \quad \leftarrow \mathtt{p} \quad \right|$$

From LP to DLP (2)

LP game + subset

$$\mathcal{P} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{a} \vee \mathtt{b} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}$$

goal : $\leftarrow \underline{\mathtt{p}}$

From LP to DLP (2)

LP game + subset

$$\mathcal{P} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{a} \vee \mathtt{b} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	

From LP to DLP (2)

LP game + subset

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From LP to DLP (2)

LP game + subset

$$\mathcal{P} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{a} \vee \mathtt{b} \leftarrow \phantom{\mathtt{a}} \end{array} \right\} \quad \left| \begin{array}{rr} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ B_0: & \mathtt{p} \leftarrow \underline{\mathtt{a}} \\ & \end{array} \right|$$

From LP to DLP (2)

LP game + subset

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From LP to DLP (2)

LP game + subset

$$\mathcal{P} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{a} \vee \mathtt{b} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	$\mathtt{p} \leftarrow \mathtt{b}$

From LP to DLP (2)

LP game + subset

$$\mathcal{P} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{a} \vee \mathtt{b} \leftarrow \phantom{\mathtt{a}} \end{array} \right\} \quad \left| \begin{array}{rr} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ \mathsf{B}_0: & \mathtt{p} \leftarrow \underline{\mathtt{b}} \\ & \end{array} \right|$$

From LP to DLP (2)

LP game + subset

$$\mathcal{P} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{a} \vee \mathtt{b} \leftarrow \phantom{\mathtt{a}} \end{array} \right\} \quad \left| \begin{array}{rr} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ B_0: & \mathtt{p} \leftarrow \mathtt{b} \\ B_1: & \ddot\frown \end{array} \right|$$

From LP to DLP (2)

(2) Combining rules

Believer is allowed to use more than one rule (**combo** move).

Meaning of combination

Notice, that if we accept the two rules

$$H \leftarrow B \qquad \text{and} \qquad H' \leftarrow B'$$

then their *combination*

$$H \vee H' \leftarrow B \vee B'$$

should also be accepted.

From LP to DLP (2)

(2) Combining rules

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Notice, that if we accept the two rules

$$H \leftarrow B \qquad \text{and} \qquad H' \leftarrow B'$$

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From LP to DLP (2)

LP game + subset + combo

$$\mathcal{P} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{a} \vee \mathtt{b} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$

From LP to DLP (2)

LP game + subset + combo

$$\mathcal{P} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{a} \vee \mathtt{b} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	

From LP to DLP (2)

LP game + subset + combo

$$\mathcal{P} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ a \vee b \leftarrow \end{array} \right\}$$

goal :	$\leftarrow \underline{p}$
B_0 :	$(p \leftarrow a)$
	$(p \leftarrow b)$

From LP to DLP (2)

LP game + subset + combo

$$\mathcal{P} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{a} \vee \mathtt{b} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	$(\mathtt{p} \leftarrow \mathtt{a})$
	$(\mathtt{p} \leftarrow \mathtt{b})$
	$\mathtt{p} \leftarrow \mathtt{a} \vee \mathtt{b}$

From LP to DLP (2)

LP game + subset + combo

$$\mathcal{P} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ a \vee b \leftarrow \end{array} \right\}$$

goal :	$\leftarrow \underline{p}$
B_0 :	$(p \leftarrow a)$
	$(p \leftarrow b)$
	$p \leftarrow a \vee b$

From LP to DLP (2)

LP game + subset + combo

$$\mathcal{P} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{a} \vee \mathtt{b} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}$$

$$\left| \begin{array}{rl} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ B_0: & (\mathtt{p} \leftarrow \mathtt{a}) \\ & (\mathtt{p} \leftarrow \mathtt{b}) \\ & \mathtt{p} \leftarrow \underline{\mathtt{a} \vee \mathtt{b}} \\ B_1: & \end{array} \right|$$

From LP to DLP (2)

LP game + subset + combo

$$\mathcal{P} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{a} \vee \mathtt{b} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}$$

$$\begin{array}{|rl|} \text{goal}: & \qquad \leftarrow \underline{\mathtt{p}} \\ B_0: & \quad (\mathtt{p} \leftarrow \mathtt{a}) \\ & \quad (\mathtt{p} \leftarrow \mathtt{b}) \\ & \quad \mathtt{p} \leftarrow \underline{\mathtt{a} \vee \mathtt{b}} \\ B_1: & \mathtt{a} \vee \mathtt{b} \leftarrow \end{array}$$

From LP to DLP (2)

LP game + subset + combo

$$\mathcal{P} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ a \vee b \leftarrow \end{array} \right\}$$

$$\left| \begin{array}{lr} \text{goal}: & \leftarrow \underline{p} \\ B_0: & (p \leftarrow a) \\ & (p \leftarrow b) \\ & p \leftarrow \underline{a \vee b} \\ B_1: & a \vee b \leftarrow \emptyset \end{array} \right|$$

Believer wins! $\ddot\smile$

From LP to DLP (3)

LP game + subset + combo

$$\mathcal{Q} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{b} \leftarrow \mathtt{c} \\ \mathtt{a} \vee \mathtt{c} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}$$

goal : $\leftarrow \mathtt{p}$

From LP to DLP (3)

LP game + subset + combo

$$\mathcal{Q} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{b} \leftarrow \mathtt{c} \\ \mathtt{a} \vee \mathtt{c} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}$$

goal : $\leftarrow \mathtt{p}$

From LP to DLP (3)

LP game + subset + combo

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

goal : $\leftarrow \underline{p}$

From LP to DLP (3)

LP game + subset + combo

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

goal :	$\leftarrow \underline{p}$

From LP to DLP (3)

LP game + subset + combo

$$\mathcal{Q} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{b} \leftarrow \mathtt{c} \\ \mathtt{a} \vee \mathtt{c} \leftarrow \phantom{\mathtt{c}} \end{array} \right\}$$

$$\begin{array}{|rc|} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ B_0: & (\mathtt{p} \leftarrow \mathtt{a}) \\ & (\mathtt{p} \leftarrow \mathtt{b}) \\ \end{array}$$

From LP to DLP (3)

LP game + subset + combo

$$\mathcal{Q} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{b} \leftarrow \mathtt{c} \\ \mathtt{a} \vee \mathtt{c} \leftarrow \phantom{\mathtt{c}} \end{array} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	$(\mathtt{p} \leftarrow \mathtt{a})$
	$(\mathtt{p} \leftarrow \mathtt{b})$
	$\mathtt{p} \leftarrow \mathtt{a} \vee \mathtt{b}$

From LP to DLP (3)

LP game + subset + combo

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

goal :	$\leftarrow \underline{p}$
B_0 :	$(p \leftarrow a)$
	$(p \leftarrow b)$
	$p \leftarrow a \vee b$

From LP to DLP (3)

LP game + subset + combo

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

goal :	$\leftarrow \underline{p}$
B_0 :	$(p \leftarrow a)$
	$(p \leftarrow b)$
	$p \leftarrow \underline{a \vee b}$

From LP to DLP (3)

LP game + subset + combo

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

$$\left| \begin{array}{ll} \text{goal}: & \leftarrow \underline{p} \\ B_0: & (p \leftarrow a) \\ & (p \leftarrow b) \\ & p \leftarrow \underline{a \vee b} \\ B_1: & b \leftarrow c \end{array} \right|$$

From LP to DLP (3)

LP game + subset + combo

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

$$\left| \begin{array}{ll} \text{goal}: & \leftarrow \underline{p} \\ B_0: & (p \leftarrow a) \\ & (p \leftarrow b) \\ & p \leftarrow \underline{a \vee b} \\ B_1: & b \leftarrow \textcolor{red}{c} \end{array} \right|$$

From LP to DLP (3)

LP game + subset + combo

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

$$\left| \begin{array}{rl} \text{goal}: & \leftarrow \underline{p} \\ B_0: & (p \leftarrow a) \\ & (p \leftarrow b) \\ & p \leftarrow \underline{a \vee b} \\ B_1: & b \leftarrow \underline{c} \end{array} \right|$$

From LP to DLP (3)

LP game + subset + combo

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

$$\left| \begin{array}{ll} \text{goal}: & \leftarrow \underline{p} \\ B_0: & (p \leftarrow a) \\ & (p \leftarrow b) \\ & p \leftarrow \underline{a \vee b} \\ B_1: & b \leftarrow \underline{c} \end{array} \right|$$

Believer lost! $\ddot{\frown}$

From LP to DLP (3)

(3) Implicit rules

Believer is allowed to use *implicit* rules of the form

$$\mathtt{a} \leftarrow \mathtt{a},$$

not from the program.

From LP to DLP (3)

LP game + subset + combo + implicit

$$\{a \leftarrow a, \quad b \leftarrow b, \quad c \leftarrow c, \quad p \leftarrow p\}$$

$$\mathcal{Q} \coloneqq \left\{\begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array}\right\}$$

goal : $\leftarrow \underline{p}$

From LP to DLP (3)

LP game + subset + combo + implicit

$$\{a \leftarrow a, \quad b \leftarrow b, \quad c \leftarrow c, \quad p \leftarrow p\}$$

$$\mathcal{Q} \coloneqq \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

goal :	$\leftarrow \underline{p}$
B_0 :	

From LP to DLP (3)

LP game + subset + combo + implicit

$$\{a \leftarrow a, \quad b \leftarrow b, \quad c \leftarrow c, \quad p \leftarrow p\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

goal :	$\leftarrow \underline{p}$
B_0 :	$p \leftarrow a \vee b$

From LP to DLP (3)

LP game + subset + combo + implicit

$$\{a \leftarrow a, \quad b \leftarrow b, \quad c \leftarrow c, \quad p \leftarrow p\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

goal :	$\leftarrow \underline{p}$
B_0 :	$p \leftarrow a \vee b$

From LP to DLP (3)

LP game + subset + combo + implicit

$$\{a \leftarrow a, \quad b \leftarrow b, \quad c \leftarrow c, \quad p \leftarrow p\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

goal :	$\leftarrow \underline{p}$
B_0 :	$p \leftarrow \underline{a \vee b}$

From LP to DLP (3)

LP game + subset + combo + implicit

$$\{\mathtt{a} \leftarrow \mathtt{a}, \quad \mathtt{b} \leftarrow \mathtt{b}, \quad \mathtt{c} \leftarrow \mathtt{c}, \quad \mathtt{p} \leftarrow \mathtt{p}\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{b} \leftarrow \mathtt{c} \\ \mathtt{a} \vee \mathtt{c} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	$\mathtt{p} \leftarrow \underline{\mathtt{a} \vee \mathtt{b}}$
B_1 :	

From LP to DLP (3)

LP game + subset + combo + implicit

$$\{a \leftarrow a, \quad b \leftarrow b, \quad c \leftarrow c, \quad p \leftarrow p\}$$

$$\mathcal{Q} \coloneqq \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

goal :	$\leftarrow \underline{p}$
B_0 :	$p \leftarrow \underline{a \vee b}$
B_1 :	$a \vee b \leftarrow a \vee c$

From LP to DLP (3)

LP game + subset + combo + implicit

$$\{a \leftarrow a, \quad b \leftarrow b, \quad c \leftarrow c, \quad p \leftarrow p\}$$

$$\mathcal{Q} \coloneqq \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

$$\begin{array}{rr} \text{goal}: & \leftarrow \underline{p} \\ B_0: & p \leftarrow \underline{a \vee b} \\ B_1: & a \vee b \leftarrow a \vee c \end{array}$$

From LP to DLP (3)

LP game + subset + combo + implicit

$$\{a \leftarrow a, \quad b \leftarrow b, \quad c \leftarrow c, \quad p \leftarrow p\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

$$\begin{array}{|rr|} \text{goal}: & \leftarrow \underline{p} \\ B_0: & p \leftarrow \underline{a \vee b} \\ B_1: & a \vee b \leftarrow \underline{a \vee c} \\ & \end{array}$$

From LP to DLP (3)

LP game + subset + combo + implicit

$$\{a \leftarrow a, \quad b \leftarrow b, \quad c \leftarrow c, \quad p \leftarrow p\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}
\qquad
\begin{array}{|rr|}
\text{goal}: & \leftarrow \underline{p} \\
B_0: & p \leftarrow \underline{a \vee b} \\
B_1: & a \vee b \leftarrow \underline{a \vee c} \\
B_2: & \\
\end{array}$$

From LP to DLP (3)

LP game + subset + combo + implicit

$$\{a \leftarrow a, \quad b \leftarrow b, \quad c \leftarrow c, \quad p \leftarrow p\}$$

$$\mathcal{Q} \coloneqq \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\} \quad \left| \begin{array}{rr} \text{goal}: & \leftarrow \underline{p} \\ B_0: & p \leftarrow \underline{a \vee b} \\ B_1: & a \vee b \leftarrow \underline{a \vee c} \\ B_2: & a \vee c \leftarrow \end{array} \right|$$

From LP to DLP (3)

LP game + subset + combo + implicit

$$\{a \leftarrow a, \quad b \leftarrow b, \quad c \leftarrow c, \quad p \leftarrow p\}$$

$$\mathcal{Q} \coloneqq \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\} \qquad \begin{array}{|lr|} \text{goal}: & \leftarrow \underline{p} \\ B_0: & p \leftarrow \underline{a \vee b} \\ B_1: & a \vee b \leftarrow \underline{a \vee c} \\ B_2: & a \vee c \leftarrow \emptyset \end{array}$$

Believer wins! $\ddot\smile$

The DLP game

Remark: what do we really need?

$$\text{DLP game} \stackrel{?}{=} \text{LP game} + \text{ subset } + \text{combo} + \text{implicit}$$

The DLP game

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$$\text{DLP game} \stackrel{?}{=} \text{LP game} + (\text{subset}) + \text{combo} + \text{implicit}$$

We can obtain the "subset" by "combo" and "implicit".

The DLP game

Remark: what do we really need?

$$\text{DLP game} \stackrel{?}{=} \text{LP game} + \text{combo} + \text{implicit}$$

We can obtain the "subset" by "combo" and "implicit".

The DLP game

Remark: what do we really need?

$$\text{DLP game} \stackrel{?}{=} \text{LP game} + \text{combo} + \text{implicit}$$

The DLP game

Finally...

$$\text{DLP game} = \text{LP game} + \text{combo} + \text{implicit}$$

The DLPG game

an example of stalling

$$\{\mathtt{a} \leftarrow \mathtt{a}, \quad \mathtt{b} \leftarrow \mathtt{b}, \quad \mathtt{c} \leftarrow \mathtt{c}, \quad \mathtt{p} \leftarrow \mathtt{p}\}$$

$$\mathcal{Q} \coloneqq \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{b} \leftarrow \mathtt{c} \\ \mathtt{a} \vee \mathtt{c} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}$$

goal : $\leftarrow \underline{\mathtt{p}}$

The DLPG game

an example of stalling

$$\{a \leftarrow a, \quad b \leftarrow b, \quad c \leftarrow c, \quad p \leftarrow p\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

goal : $\leftarrow \underline{p}$

B_0 :

The DLPG game

an example of stalling

$$\{\mathtt{a} \leftarrow \mathtt{a}, \quad \mathtt{b} \leftarrow \mathtt{b}, \quad \mathtt{c} \leftarrow \mathtt{c}, \quad \mathtt{p} \leftarrow \mathtt{p}\}$$

$$\mathcal{Q} \coloneqq \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{b} \leftarrow \mathtt{c} \\ \mathtt{a} \vee \mathtt{c} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	$\mathtt{p} \leftarrow \mathtt{a} \vee \mathtt{b}$

The DLPG game

an example of stalling

$$\{\mathtt{a} \leftarrow \mathtt{a}, \quad \mathtt{b} \leftarrow \mathtt{b}, \quad \mathtt{c} \leftarrow \mathtt{c}, \quad \mathtt{p} \leftarrow \mathtt{p}\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{b} \leftarrow \mathtt{c} \\ \mathtt{a} \vee \mathtt{c} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	$\mathtt{p} \leftarrow \mathtt{a} \vee \mathtt{b}$

The DLPG game

an example of stalling

$$\{\mathtt{a} \leftarrow \mathtt{a}, \quad \mathtt{b} \leftarrow \mathtt{b}, \quad \mathtt{c} \leftarrow \mathtt{c}, \quad \mathtt{p} \leftarrow \mathtt{p}\}$$

$$\mathcal{Q} \coloneqq \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{b} \leftarrow \mathtt{c} \\ \mathtt{a} \vee \mathtt{c} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	$\mathtt{p} \leftarrow \underline{\mathtt{a} \vee \mathtt{b}}$

The DLPG game

an example of stalling

$$\{a \leftarrow a, \quad b \leftarrow b, \quad c \leftarrow c, \quad p \leftarrow p\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

goal :	$\leftarrow \underline{p}$
B_0 :	$p \leftarrow \underline{a \vee b}$
B_1 :	

The DLPG game

an example of stalling

$$\{\mathtt{a} \leftarrow \mathtt{a}, \quad \mathtt{b} \leftarrow \mathtt{b}, \quad \mathtt{c} \leftarrow \mathtt{c}, \quad \mathtt{p} \leftarrow \mathtt{p}\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{b} \leftarrow \mathtt{c} \\ \mathtt{a} \vee \mathtt{c} \leftarrow \quad \end{array} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	$\mathtt{p} \leftarrow \underline{\mathtt{a} \vee \mathtt{b}}$
B_1 :	$\mathtt{a} \vee \mathtt{b} \leftarrow \mathtt{a} \vee \mathtt{b}$

Believer is stalling. . .

The DLPG game

an example of stalling

$$\{\mathtt{a} \leftarrow \mathtt{a}, \quad \mathtt{b} \leftarrow \mathtt{b}, \quad \mathtt{c} \leftarrow \mathtt{c}, \quad \mathtt{p} \leftarrow \mathtt{p}\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{b} \leftarrow \mathtt{c} \\ \mathtt{a} \vee \mathtt{c} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}$$

$$\begin{array}{lr} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ \mathsf{B}_0: & \mathtt{p} \leftarrow \underline{\mathtt{a} \vee \mathtt{b}} \\ \mathsf{B}_1: & \mathtt{a} \vee \mathtt{b} \leftarrow \mathtt{a} \vee \mathtt{b} \end{array}$$

Believer is stalling. . .

The DLPG game

an example of stalling

$$\{\mathtt{a} \leftarrow \mathtt{a}, \quad \mathtt{b} \leftarrow \mathtt{b}, \quad \mathtt{c} \leftarrow \mathtt{c}, \quad \mathtt{p} \leftarrow \mathtt{p}\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{b} \leftarrow \mathtt{c} \\ \mathtt{a} \lor \mathtt{c} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}$$

$$\begin{array}{|lr|} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ B_0: & \mathtt{p} \leftarrow \underline{\mathtt{a} \lor \mathtt{b}} \\ B_1: & \mathtt{a} \lor \mathtt{b} \leftarrow \underline{\mathtt{a} \lor \mathtt{b}} \end{array}$$

Believer is stalling. . .

The DLPG game

an example of stalling

$$\{a \leftarrow a, \quad b \leftarrow b, \quad c \leftarrow c, \quad p \leftarrow p\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

goal :	$\leftarrow \underline{p}$
B_0 :	$p \leftarrow \underline{a \vee b}$
B_1 :	$a \vee b \leftarrow \underline{a \vee b}$
B_2 :	

Believer is stalling. . .

The DLPG game

an example of stalling

$$\{\mathtt{a} \leftarrow \mathtt{a}, \quad \mathtt{b} \leftarrow \mathtt{b}, \quad \mathtt{c} \leftarrow \mathtt{c}, \quad \mathtt{p} \leftarrow \mathtt{p}\}$$

$$\mathcal{Q} \coloneqq \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{b} \leftarrow \mathtt{c} \\ \mathtt{a} \vee \mathtt{c} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	$\mathtt{p} \leftarrow \underline{\mathtt{a} \vee \mathtt{b}}$
B_1 :	$\mathtt{a} \vee \mathtt{b} \leftarrow \underline{\mathtt{a} \vee \mathtt{b}}$
B_2 :	$\mathtt{a} \vee \mathtt{b} \leftarrow \mathtt{a} \vee \mathtt{b}$

Believer is stalling. . .

The DLPG game

an example of stalling

$$\{a \leftarrow a, \quad b \leftarrow b, \quad c \leftarrow c, \quad p \leftarrow p\}$$

$$\mathcal{Q} \coloneqq \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

goal :	$\leftarrow \underline{p}$
B_0 :	$p \leftarrow \underline{a \vee b}$
B_1 :	$a \vee b \leftarrow \underline{a \vee b}$
B_2 :	$a \vee b \leftarrow a \vee b$

Believer is stalling. . .

The DLPG game

an example of stalling

$$\{\mathtt{a} \leftarrow \mathtt{a}, \quad \mathtt{b} \leftarrow \mathtt{b}, \quad \mathtt{c} \leftarrow \mathtt{c}, \quad \mathtt{p} \leftarrow \mathtt{p}\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{b} \leftarrow \mathtt{c} \\ \mathtt{a} \vee \mathtt{c} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}
\quad
\left| \begin{array}{rr} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ B_0: & \mathtt{p} \leftarrow \underline{\mathtt{a} \vee \mathtt{b}} \\ B_1: & \mathtt{a} \vee \mathtt{b} \leftarrow \underline{\mathtt{a} \vee \mathtt{b}} \\ B_2: & \mathtt{a} \vee \mathtt{b} \leftarrow \underline{\mathtt{a} \vee \mathtt{b}} \end{array} \right|$$

Believer is stalling. . .

The DLPG game

an example of stalling

$$\{a \leftarrow a, \quad b \leftarrow b, \quad c \leftarrow c, \quad p \leftarrow p\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \quad \end{array} \right\}$$

$$\begin{array}{|rr|} \text{goal}: & \leftarrow \underline{p} \\ B_0: & p \leftarrow \underline{a \vee b} \\ B_1: & a \vee b \leftarrow \underline{a \vee b} \\ B_2: & a \vee b \leftarrow \underline{a \vee b} \\ B_3: & \\ \end{array}$$

Believer is stalling. . .

The DLPG game

an example of stalling

$$\{\mathtt{a} \leftarrow \mathtt{a}, \quad \mathtt{b} \leftarrow \mathtt{b}, \quad \mathtt{c} \leftarrow \mathtt{c}, \quad \mathtt{p} \leftarrow \mathtt{p}\}$$

$$\mathcal{Q} \coloneqq \left\{ \begin{aligned} \mathtt{p} &\leftarrow \mathtt{a} \\ \mathtt{p} &\leftarrow \mathtt{b} \\ \mathtt{b} &\leftarrow \mathtt{c} \\ \mathtt{a} \vee \mathtt{c} &\leftarrow \end{aligned} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	$\mathtt{p} \leftarrow \underline{\mathtt{a} \vee \mathtt{b}}$
B_1 :	$\mathtt{a} \vee \mathtt{b} \leftarrow \underline{\mathtt{a} \vee \mathtt{b}}$
B_2 :	$\mathtt{a} \vee \mathtt{b} \leftarrow \underline{\mathtt{a} \vee \mathtt{b}}$
B_3 :	$\mathtt{a} \vee \mathtt{b} \leftarrow \mathtt{a} \vee \mathtt{c}$

The DLPG game

an example of stalling

$$\{a \leftarrow a, \quad b \leftarrow b, \quad c \leftarrow c, \quad p \leftarrow p\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

$$\begin{array}{lr} \text{goal}: & \leftarrow \underline{p} \\ B_0: & p \leftarrow \underline{a \vee b} \\ B_1: & a \vee b \leftarrow \underline{a \vee b} \\ B_2: & a \vee b \leftarrow \underline{a \vee b} \\ B_3: & a \vee b \leftarrow {\color{red} a \vee c} \end{array}$$

The DLPG game

an example of stalling

$$\{\mathtt{a} \leftarrow \mathtt{a}, \quad \mathtt{b} \leftarrow \mathtt{b}, \quad \mathtt{c} \leftarrow \mathtt{c}, \quad \mathtt{p} \leftarrow \mathtt{p}\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{b} \leftarrow \mathtt{c} \\ \mathtt{a} \vee \mathtt{c} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}$$

$$\begin{array}{|rr|} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ B_0: & \mathtt{p} \leftarrow \underline{\mathtt{a} \vee \mathtt{b}} \\ B_1: & \mathtt{a} \vee \mathtt{b} \leftarrow \underline{\mathtt{a} \vee \mathtt{b}} \\ B_2: & \mathtt{a} \vee \mathtt{b} \leftarrow \underline{\mathtt{a} \vee \mathtt{b}} \\ B_3: & \mathtt{a} \vee \mathtt{b} \leftarrow \underline{\mathtt{a} \vee \mathtt{c}} \end{array}$$

The DLPG game

an example of stalling

$$\{a \leftarrow a, \quad b \leftarrow b, \quad c \leftarrow c, \quad p \leftarrow p\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

$$\left| \begin{array}{rr} \text{goal}: & \leftarrow \underline{p} \\ B_0: & p \leftarrow \underline{a \vee b} \\ B_1: & a \vee b \leftarrow \underline{a \vee b} \\ B_2: & a \vee b \leftarrow \underline{a \vee b} \\ B_3: & a \vee b \leftarrow \underline{a \vee c} \\ B_4: & \end{array} \right|$$

The DLPG game

an example of stalling

$$\{\mathtt{a} \leftarrow \mathtt{a}, \quad \mathtt{b} \leftarrow \mathtt{b}, \quad \mathtt{c} \leftarrow \mathtt{c}, \quad \mathtt{p} \leftarrow \mathtt{p}\}$$

$$\mathcal{Q} \coloneqq \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{b} \leftarrow \mathtt{c} \\ \mathtt{a} \vee \mathtt{c} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}$$

$$\left| \begin{array}{lr} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ B_0: & \mathtt{p} \leftarrow \underline{\mathtt{a} \vee \mathtt{b}} \\ B_1: & \mathtt{a} \vee \mathtt{b} \leftarrow \underline{\mathtt{a} \vee \mathtt{b}} \\ B_2: & \mathtt{a} \vee \mathtt{b} \leftarrow \underline{\mathtt{a} \vee \mathtt{b}} \\ B_3: & \mathtt{a} \vee \mathtt{b} \leftarrow \underline{\mathtt{a} \vee \mathtt{c}} \\ B_4: & \mathtt{a} \vee \mathtt{c} \leftarrow \phantom{\mathtt{a} \vee \mathtt{c}} \end{array} \right|$$

The DLPG game

an example of stalling

$$\{\mathtt{a} \leftarrow \mathtt{a}, \quad \mathtt{b} \leftarrow \mathtt{b}, \quad \mathtt{c} \leftarrow \mathtt{c}, \quad \mathtt{p} \leftarrow \mathtt{p}\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{b} \leftarrow \mathtt{c} \\ \mathtt{a} \vee \mathtt{c} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}
\qquad
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Believer wins! $\ddot\smile$

The DLP game

How to get semantics out of it

We look at strategies (The same as in LP!)

The value of $p_1 \vee \cdots \vee p_k$ w.r.t. $\mathcal{P}$ is determined by the game: $\leftarrow p_1 \vee \cdots \vee p_k$ succeeds iff there is a winning strategy (for the Believer).

The connecting result

Theorem (me, 2013)

$$\textbf{DLPG} \approx \textbf{MM}$$

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Soundness and completeness

A proof sketch

We are given a DLP program $\mathcal{P}$ and a goal G.

Proof is by induction on the number of $\vee$ in heads of $\mathcal{P}$.

- We select a **disjunctive** rule ϕ from $\mathcal{P}$, e.g.,

$$\underbrace{\mathtt{p} \vee \mathtt{q} \vee \mathtt{r} \leftarrow \mathtt{a}\ ,\ \mathtt{b} \vee \mathtt{c}\ ,\ \mathtt{d}}_{\phi}$$

- and we **split it in two** rules, ϕ_1 and ϕ_2 by splitting its head:

- then we look at the *"less disjunctive"* programs $\mathcal{P}_1$ and $\mathcal{P}_2$ obtained by **replacing** ϕ by ϕ_1 and ϕ_2 respectively.

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A proof sketch (cont'd)

Completeness (Minker semantics $\implies$ DLP game semantics)

- G is true in every minimal model of $\mathcal{P}$.
- We argue that it must be true in every minimal model of $\mathcal{P}_1$ and of $\mathcal{P}_2$.
- By the induction hypothesis we obtain winning strategies for $\mathcal{P}_1$ and $\mathcal{P}_2$.
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How do we combine plays?

σ_1	ρ_1, ρ_2	ρ_8, ϕ_1	ρ_2, ϕ_1	ρ_1	ρ_3	ρ_5	ρ_2	
σ_2	ρ_2, ρ_5	ϕ_2, ρ_5	ρ_3, ρ_4	ρ_6, ρ_7	ρ_3, ϕ_2	ρ_9		
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Soundness and completeness

A proof sketch (cont'd)

Soundness (DLP game semantics $\implies$ Minker semantics)

Similar.

A game semantics for LPN

The **LPNG** game

- Whenever a doubter doubts a negated atom $\sim\mathtt{p}$,
  the rôles of the players switch:
  the believer becomes the doubter, doubting $\mathtt{p}$.
- This implies draws.

What about DLPN?

A game semantics seems difficult to define directly.

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An abstract semantic framework

Defining the notions of...

- truth value space $\mathcal{V}$ (e.g., $\mathbb{B}$, $\mathbb{V}_\kappa$, ...)
  (completely distributive Heyting algebra $+ \cdots$)
- semantics of a language L as objects of study

$$\mathbf{s} \; : \; \mathbf{P}_L \times \mathbf{Q}_L \to \mathcal{V};$$

- equivalence of semantics ($\approx$);
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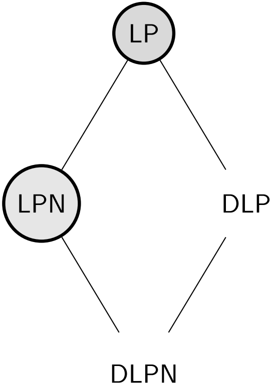
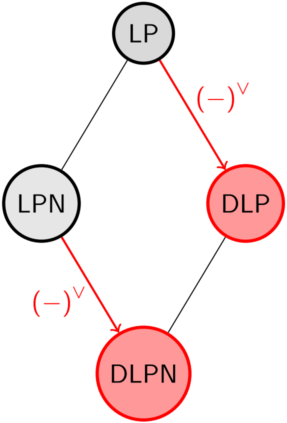
The semantic operator $(-)^\vee$



Transforming semantics

- Start with any semantics s for a non-disjunctive language.
- Apply the operator to get a new semantics $(s)^\vee$ for the corresponding disjunctive language.

The semantic operator $(-)^{\vee}$



Transforming semantics

- Start with any semantics s for a non-disjunctive language.
- Apply the operator to get a new semantics $(s)^{\vee}$ for the corresponding disjunctive language.

Properties of $(-)^{\vee}$

Preservation properties

The operator respects equivalences:

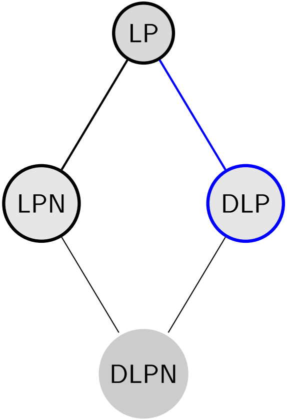
$$s_1 \approx s_2 \implies (s_1)^{\vee} \approx (s_2)^{\vee}$$

Expected outcomes

The operator yields the expected results for the standard semantics:

$$\text{(Least Herbrand Model)}^{\vee} \approx \text{Minimal Models}$$
$$\text{(Well-Founded Model)}^{\vee} \approx \infty\text{-valued Minimal Models}$$

Applications of $(-)^\vee$ on game semantics



Game semantics

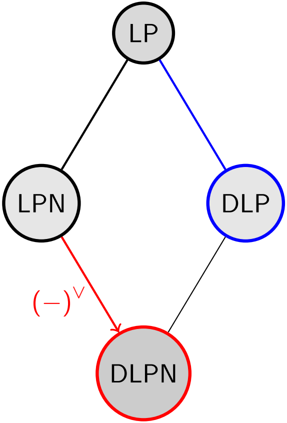
LP: Di Cosmo, Loddo & Nicolet (1998)

LPN: Rondogiannis & Wadge (2005)

DLP: me (2013)

DLPN: ?

Applications of $(-)^{\vee}$ on game semantics



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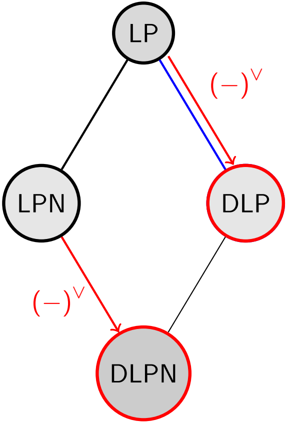

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Definite instantiations

Consider the disjunctive program

$$\mathcal{D} := \left\{ \begin{array}{rl} \mathtt{s} \vee \mathtt{t} & \leftarrow \mathtt{p}\,,\, \mathtt{b}\,,\, \mathtt{c} \\ \mathtt{a} \vee \mathtt{b} & \leftarrow \\ \mathtt{p} & \leftarrow \mathtt{a} \\ \mathtt{p} & \leftarrow \mathtt{b}\,,\, \mathtt{d}\,,\, \mathtt{f} \\ \mathtt{b} \vee \mathtt{c} & \leftarrow \end{array} \right\}.$$

The set $\mathrm{D}(\mathcal{D})$ of its definite instantiations is

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$$\mathcal{D} \coloneqq \left\{ \begin{array}{rcl} \mathtt{s} \vee \mathtt{t} & \leftarrow & \mathtt{p} \, , \, \mathtt{b} \, , \, \mathtt{c} \\ \mathtt{a} \vee \mathtt{b} & \leftarrow & \\ \mathtt{p} & \leftarrow & \mathtt{a} \\ \mathtt{p} & \leftarrow & \mathtt{b} \, , \, \mathtt{d} \, , \, \mathtt{f} \\ \mathtt{b} \vee \mathtt{c} & \leftarrow & \end{array} \right\}.$$

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Definition of $(-)^{\vee}$ for a semantics $\mathbf{s}$ of LP

$$
\begin{array}{lrl}
\text{if} & \mathbf{s} & : \ \mathbf{P}_{\mathsf{LP}} \times \mathbf{Q}_{\mathsf{LP}} \to \mathcal{V}, \\
\text{then} & (\mathbf{s})^{\vee} & : \ \mathbf{P}_{\mathsf{DLP}} \times \mathbf{Q}_{\mathsf{DLP}} \to \mathcal{V}, \\
\text{is defined by} & (\mathbf{s})^{\vee}(\mathcal{D}, G) \triangleq &
\end{array}
$$

Given...

an LP semantics $\mathbf{s}$,
a DLP program $\mathcal{D}$,
and a DLP goal $G = g_1 \vee \cdots \vee g_r$,

- we seek the truth value of G in $\mathcal{D}$
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if $\mathbf{s} : \mathbf{P}_{\mathrm{LP}} \times \mathbf{Q}_{\mathrm{LP}} \to \mathcal{V}$,

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What is $(\mathbf{LPG})^{\vee}$?

An example of $(-)^{\vee}$ in use

$$\mathcal{D} := \left\{ \begin{array}{rcl} \mathtt{s} \vee \mathtt{t} & \leftarrow & \mathtt{p}, \mathtt{b}, \mathtt{c} \\ \mathtt{a} \vee \mathtt{b} & \leftarrow & \\ \mathtt{p} & \leftarrow & \mathtt{a} \\ \mathtt{p} & \leftarrow & \mathtt{b}, \mathtt{d}, \mathtt{f} \\ \mathtt{b} \vee \mathtt{c} & \leftarrow & \end{array} \right\}$$

goal : $\leftarrow \mathtt{p} \vee \mathtt{s} \vee \mathtt{t}$

$$\mathrm{D}(\mathcal{D}) = \{\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \mathcal{P}_4, \mathcal{P}_5, \mathcal{P}_6, \mathcal{P}_7, \mathcal{P}_8\}.$$

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B_0 :	$\mathtt{t}$
goal :	$\leftarrow \mathtt{t}$

$$\mathrm{D}(\mathcal{D}) = \{\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \mathcal{P}_4, \mathcal{P}_5, \mathcal{P}_6, \mathcal{P}_7, \mathcal{P}_8\}.$$

What is $(\mathbf{LPG})^{\vee}$?

An example of $(-)^{\vee}$ in use

$$\mathcal{P}_4 := \left\{ \begin{array}{l} \mathtt{t} \leftarrow \mathtt{p}\ ,\ \mathtt{b}\ ,\ \mathtt{c} \\ \mathtt{b} \leftarrow \\ \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b}\ ,\ \mathtt{d}\ ,\ \mathtt{f} \\ \mathtt{c} \leftarrow \end{array} \right\}$$

$$\begin{array}{|rl|} \text{goal}: & \leftarrow \mathtt{p} \vee \mathtt{s} \vee \mathtt{t} \\ \mathrm{D}_0: & \mathcal{P}_4 \\ \mathrm{B}_0: & \mathtt{t} \\ \text{goal}: & \leftarrow \underline{\mathtt{t}} \\ & \end{array}$$

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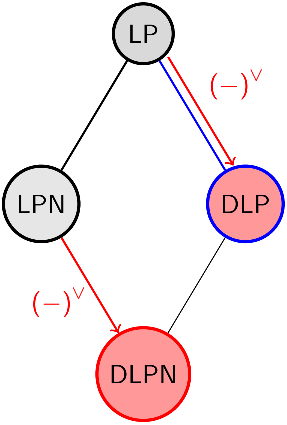
Summary & future research



Contributions

- abstract semantic framework, truth value spaces, ...
- semantic operator $(-)^{\vee}$
- "hand-made" DLP game
- infinite $\Longrightarrow$ first-order

Investigate how these ideas apply to:

- higher-order logic programming;
- coalgebraic semantics.

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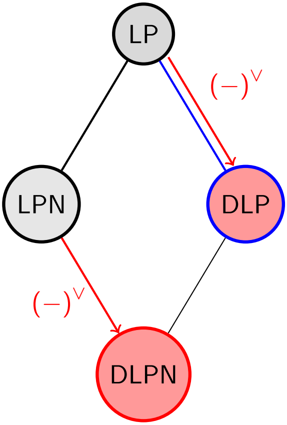

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Thanks!

Questions?

Θ.

Bonus tracks. . .

LP

Fixpoint construction of the Least Herbrand Model

$$\mathcal{P} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{a}\ ,\ \mathtt{b} \\ \mathtt{p} \leftarrow \mathtt{c} \\ \mathtt{a} \leftarrow \mathtt{e} \\ \mathtt{b} \leftarrow \mathtt{d} \\ \mathtt{b} \leftarrow \mathtt{e} \\ \mathtt{e} \leftarrow \\ \mathtt{f} \leftarrow \end{array} \right\}$$

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Least Herbrand Model of $\mathcal{P} = \{\mathtt{e}, \mathtt{f}, \mathtt{a}, \mathtt{b}, \mathtt{p}\}$

Why propositional?

Cheating conveniently

Are we forgetting something?

- Logic programs may contain variables and function symbols.
- Instead, we are only looking at (possibly infinite) propositional logic programs.

Why propositional?

Example

The program

$$\left\{\begin{array}{r}\mathtt{even}(0) \leftarrow \qquad\qquad\quad \\ \mathtt{even}(S(S(X))) \leftarrow \mathtt{even}(X) \quad \\ \mathtt{odd}(X) \leftarrow {\sim}\mathtt{even}(X)\end{array}\right\}$$

is semantically equivalent to the program

$$\{\, \mathsf{e}_0 \leftarrow \quad \} \cup \left\{\begin{array}{c} \mathsf{e}_2 \leftarrow \mathsf{e}_0 \\ \mathsf{e}_4 \leftarrow \mathsf{e}_2 \\ \vdots \end{array}\right\} \cup \left\{\begin{array}{c} \mathsf{o}_0 \leftarrow {\sim}\mathsf{e}_0 \\ \mathsf{o}_1 \leftarrow {\sim}\mathsf{e}_1 \\ \vdots \end{array}\right\}.$$

The induction base

The induction base

- For soundness, we need to make sure that the extra rules "combo" and "implicit", do not enable us to win in any games that we could not already win without them.
- The analogous result for completeness is trivial.

A needed lemma

Lemma (Inclusions)

$$\mathrm{MM}(\mathcal{P}) \subseteq \mathrm{MM}(\mathcal{P}_1) \cup \mathrm{MM}(\mathcal{P}_2) \subseteq \mathrm{HM}(\mathcal{P}).$$

Soundness and completeness

A proof sketch (cont'd)

Soundness (DLP game semantics $\Longrightarrow$ Minker semantics)

- We have a winning strategy for $\mathcal{P}$.
- We **split this strategy** in two winning strategies, one for $\mathcal{P}_1$ and one for $\mathcal{P}_2$.
- By the induction hypothesis the goal is true in every minimal model of $\mathcal{P}_1$ and of $\mathcal{P}_2$.
- We argue that it must be true in every minimal model of $\mathcal{P}$.
- This means soundness.

Soundness and completeness

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How do we split strategies?

Splitting strategies

Whenever the strategy for $\mathcal{P}$ plays the rule ϕ, we play the "restricted" rule ϕ_1 for the game of $\mathcal{P}_1$, or ϕ_2 for $\mathcal{P}_2$.
It is easy to see that this results in a valid, winning strategy.

Induction?!

But the $\vee$s might be infinite!

Completeness. (Compactness to the rescue!)

Assume G true in every m.m. of $\mathcal{P}$. Then $\mathcal{P} \models G$, and...

$$\begin{aligned}\mathcal{P} \models G &\implies (\exists \mathcal{P}_G \subseteq_{\mathrm{fin}} \mathcal{P})[\mathcal{P}_G \models G] \\ &\implies \exists \text{ winning strategy for } \Gamma_{\mathcal{P}_G}(\leftarrow G) \\ &\qquad\qquad \text{which is also winning for } \Gamma_{\mathcal{P}}(\leftarrow G).\end{aligned}$$

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Soundness.

$\exists$ winning σ for $\Gamma_{\mathcal{P}}(\leftarrow \mathtt{G})$ $\implies$ it uses a $\mathcal{P}_\sigma \subseteq_{\text{fin}} \mathcal{P}$

$\implies$ it is also winning in $\Gamma_{\mathcal{P}_\sigma}(\leftarrow \mathtt{G})$

$\implies$ G true in every m.m. of $\mathcal{P}_\sigma$

$\implies$ G true in every m.m. of $\mathcal{P} \supseteq \mathcal{P}_\sigma$ (no negation).

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