FUNTP, 2017.1
(Turma do Thanos)

Prova escrita
(points: 256/100; Bonus: $4^\flat$; time: 100')

Nome:

14/06/2017

Regras:

I. Não vires esta página antes do começo da prova.

II. Nenhuma consulta de qualquer forma.

III. Nenhum aparelho ligado (por exemplo: celular, tablet, notebook, *etc.*).[1]

IV. Nenhuma comunicação de qualquer forma e para qualquer motivo.

V. $\forall x \big(\mathrm{Colar}(x) \to \neg \mathrm{Passar}(x, \mathrm{FUNTP})\big)$.[2]

VI. Use caneta para tuas respostas.

VII. Responda dentro das caixas indicadas.

VIII. Escreva teu nome em *cada* folha de rascunho extra, antes de usá-la.

IX. Entregue *todas* as folhas de rascunho extra, juntas com tua prova.

X. Nenhuma prova será aceita depois do fim do tempo.

XI. Os pontos bônus serão considerados apenas para quem conseguir passar sem.[3]

XII. **Escolha:**
Tem 4 letras (**A**, **B**, **C**, **D**). Cada letra **L** tem 3 problemas (**L1**, **L2**, **L3**).

(1) Se usar problema não resolvido para resolver outro, tu ganharás metade dos pontos.

(2) Para ganhar os pontos de $n+1$ problemas duma letra precisas ter ganhado os pontos de pelo menos n problemas de pelo menos $3-n$ das outras letras.

($4^\flat$) ☐ entendi.

Boas provas!

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[1] Ou seja, *desligue antes* da prova.

[2] Se essa regra não faz sentido, melhor desistir desde já.

[3] Por exemplo, 25 pontos bonus podem aumentar uma nota de 5,2 para 7,7 ou de 9,2 para 10,0, mas de 4,9 nem para 7,4 nem para 5,0. A 4,9 ficaria 4,9 mesmo.

Sequent calculus

- **LK**: either side of $\vdash$ can have any number of formulæ;
- **LJ**: the right side of $\vdash$ can have at most one formula.

$$\frac{}{A \vdash A}\ \mathrm{ax} \qquad \frac{\Gamma \vdash \Delta, D \qquad D, \Pi \vdash \Lambda}{\Gamma, \Pi \vdash \Delta, \Lambda}\ \mathrm{cut}$$

$$\frac{\Gamma \vdash \Delta}{D, \Gamma \vdash \Delta}\ \mathrm{wL} \qquad \frac{\Gamma \vdash \Delta}{\Gamma \vdash \Delta, D}\ \mathrm{wR}$$

$$\frac{D, D, \Gamma \vdash \Delta}{D, \Gamma \vdash \Delta}\ \mathrm{cL} \qquad \frac{\Gamma \vdash \Delta, D, D}{\Gamma \vdash \Delta, D}\ \mathrm{cR}$$

$$\frac{\Gamma, C, D, \Pi \vdash \Delta}{\Gamma, D, C, \Pi \vdash \Delta}\ \mathrm{xL} \qquad \frac{\Gamma \vdash \Delta, C, D, \Lambda}{\Gamma \vdash \Delta, D, C, \Lambda}\ \mathrm{xR}$$

$$\frac{\Gamma \vdash \Delta, D}{\neg D, \Gamma \vdash \Delta}\ \neg\mathrm{L} \qquad \frac{D, \Gamma \vdash \Delta}{\Gamma \vdash \Delta, \neg D}\ \neg\mathrm{R}$$

$$\frac{\Gamma \vdash \Delta, C \qquad D, \Pi \vdash \Lambda}{C \supset D, \Gamma, \Pi \vdash \Delta, \Lambda}\ \supset\mathrm{L} \qquad \frac{C, \Gamma \vdash \Delta, D}{\Gamma \vdash \Delta, C \supset D}\ \supset\mathrm{R}$$

$$\frac{C_i, \Gamma \vdash \Delta}{C_1 \wedge C_2, \Gamma \vdash \Delta}\ \wedge\mathrm{L}_i \qquad \frac{\Gamma \vdash \Delta, C \qquad \Gamma \vdash \Delta, D}{\Gamma \vdash \Delta, C \wedge D}\ \wedge\mathrm{R}$$

$$\frac{C, \Gamma \vdash \Delta \qquad D, \Gamma \vdash \Delta}{C \vee D, \Gamma \vdash \Delta}\ \vee\mathrm{L} \qquad \frac{\Gamma \vdash \Delta, C_i}{\Gamma \vdash \Delta, C_1 \vee C_2}\ \vee\mathrm{R}_i$$

(66) A. Propositional calculus

Hilbert-style deduction. The axiomatic schemes:

$$A \supset B \supset A \tag{H1}$$
$$(A \supset B \supset C) \supset (A \supset B) \supset (A \supset C) \tag{H2}$$
$$(\neg A \supset B) \supset (\neg A \supset \neg B) \supset A \tag{H3}$$

Inference rule: Modus Ponens (M.P.): from $A \supset B$ and A infer B.

Deduction theorem. $\Phi, A \vdash B \implies \Phi \vdash A \supset B$

Soundness theorem. $\Phi \vdash A \implies \Phi \models A$

(16) **A1.** Without using the deduction theorem, derive for any formula A:

$$\vdash A \supset A.$$

(28) **A2.** Prove the deduction theorem.

(22) **A3.** Prove the soundness theorem.

(52) B. Combinatory logic

Here are some combinators:

$$\begin{array}{llll}
\mathsf{I}\, x \rhd x & \mathsf{B}\, x\, y\, z \rhd x\,(y\, z) & \mathsf{S}\, x\, y\, z \rhd x\, z\,(y\, z) & \mathsf{R}\, x\, y\, z \rhd y\, z\, x \\
\mathsf{K}\, x\, y \rhd x & \mathsf{C}\, x\, y\, z \rhd x\, z\, y & \mathsf{W}\, x\, y \rhd x\, y\, y & \mathsf{V}\, x\, y\, z \rhd z\, x\, y \\
\mathsf{M}\, x \rhd x\, x & \mathsf{B}'\, x\, y\, z \rhd y\,(x\, z) & &
\end{array}$$

(14) **B1.** Define B' from I, M, B, C.

(16) **B2.** Define R from I, K, M, B, C, S, W.

(22) **B3.** Define V from I, K, M, B, C, S, W.

(70) C. Lambda calculus

Church–Rosser theorem. Suppose that for a given λ-term M we have $M \twoheadrightarrow_\beta N_1$ and $M \twoheadrightarrow_\beta N_2$. Then there is a λ-term N such that $N_1 \twoheadrightarrow_\beta N$ and $N_2 \twoheadrightarrow_\beta N$.

Corollary. *Suppose that $M =_\beta N$. Then there is some L such that $M \twoheadrightarrow_\beta L$ and $N \twoheadrightarrow_\beta L$.*

Definition. The Church numerals in λ-calculus:

$$\begin{aligned} \underline{0} &:= \lambda f\, x\,.\, x \\ \underline{1} &:= \lambda f\, x\,.\, f\, x \\ \underline{2} &:= \lambda f\, x\,.\, f\,(f\, x) \\ &\ \vdots \end{aligned}$$

(24) **C1.** Define the terms $\underline{\texttt{suc}}$, $\underline{\texttt{add}}$, $\underline{\texttt{mul}}$.

(18) **C2.** Construct a λ-term M such that

$$M\, x\, y\, z =_\beta x\, y\, z\, M.$$

(28) **C3.** Prove the corollary from the Church–Rosser theorem.

(68) D. Sequent calculus

(20) **D1.** Derive one of the following in **LJ**, and the other one in **LK** (choose wisely!):

$$\vdash A \lor \neg A \qquad\qquad \vdash \neg(\neg A \land A).$$

(24) **D2.** In **LK** derive **two** of Hilbert's axioms:

$$\vdash A \supset B \supset A \tag{H1}$$
$$\vdash (A \supset B \supset C) \supset (A \supset B) \supset (A \supset C) \tag{H2}$$
$$\vdash (\neg A \supset B) \supset (\neg A \supset \neg B) \supset A \tag{H3}$$

(24) **D3.** In **LK** derive the following:

$$\vdash \big((A \supset B) \supset A\big) \supset A. \tag{PL}$$

Dica: *This is Peirce's law, which is not intuitionistically valid.*

Só isso mesmo.

Resolução

Resolução

Resolução

Resolução

Rascunho

RASCUNHO

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