On the semantics of disjunctive logic programs

— Part I —

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Programming languages: syntax and semantics

Syntax

How programs are written.

Semantics

What programs mean.

- Denotational semantics
- Operational semantics
- Axiomatic semantics
- ...

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Programming languages: paradigms

Imperative

Describe *how* to solve the problem.
Describe *how* the program computes the solution.

vs.

Declarative

Describe *what* the problem is.
Describe *what* is a solution.

Programming languages: paradigms

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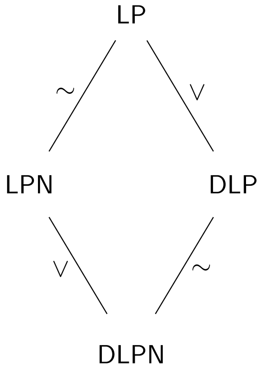
vs.

Declarative: Logic programming

Describe *what* the problem is.
Describe *what* is a solution.

Four languages: the big picture



Examples of logic programs

Example of LP

$$\left\{\begin{array}{r} \texttt{sleeps} \leftarrow \texttt{tired} \\ \texttt{works} \leftarrow \texttt{rested} \\ \texttt{eats} \leftarrow \texttt{rested}\ ,\ \texttt{hungry} \\ \texttt{rested} \leftarrow \end{array}\right\}$$

Queries

User:	$\leftarrow$ works	$\leftarrow$ tired	$\leftarrow$ eats
System:	Yes.	No.	No.

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Example of LPN

$$\left\{ \begin{array}{r} \texttt{sleeps} \leftarrow \texttt{tired} \\ \texttt{works} \leftarrow {\sim}\texttt{tired} \\ \texttt{eats} \leftarrow {\sim}\texttt{tired}\ ,\ \texttt{hungry} \end{array} \right\}$$

Queries

U:	← sleeps	← works
S:	No.	Yes.

Examples of logic programs

Example of LPN

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Queries

U:	← sleeps	← works
S:	No.	Yes.

Examples of logic programs

Example of DLP

$$\left\{\begin{aligned} \texttt{mathematician} &\leftarrow \texttt{topologist} \\ \texttt{mathematician} &\leftarrow \texttt{algebraist} \\ \texttt{algebraist} \vee \texttt{topologist} &\leftarrow \end{aligned}\right\}$$

Queries

	$\leftarrow$ algebraist	$\leftarrow$ topologist	$\leftarrow$ mathematician
U:	$\leftarrow$ algebraist	$\leftarrow$ topologist	$\leftarrow$ mathematician
S:	No.	No.	Yes.

Examples of logic programs

Example of DLP

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Queries

U:	$\leftarrow \texttt{algebraist}$	$\leftarrow \texttt{topologist}$	$\leftarrow \texttt{mathematician}$
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Queries

U:	$\leftarrow$ `algebraist`	$\leftarrow$ `topologist`	$\leftarrow$ `mathematician`
S:	`No.`	`No.`	`Yes.`

First-order logic programs

How first-order programs look like

$$\left\{\begin{array}{rl}
\texttt{father}(X, Y) & \leftarrow \texttt{male}(X), \texttt{parent}(X, Y) \\
\texttt{mother}(X, Y) & \leftarrow \texttt{female}(X), \texttt{parent}(X, Y) \\
\texttt{grandparent}(X, Z) & \leftarrow \texttt{parent}(X, Y), \texttt{parent}(Y, Z) \\
\texttt{male}(\mathit{homer}) & \leftarrow \\
\texttt{female}(\mathit{marge}) & \leftarrow \\
\texttt{parent}(\mathit{homer}, \mathit{bart}) & \leftarrow \\
\texttt{parent}(\mathit{marge}, \mathit{lisa}) & \leftarrow \\
& \vdots
\end{array}\right\}$$

First-order logic programs

finite vs infinite; propositional vs first-order

Propositional programs	**First-order programs**
finite	
infinite	

From now on...

We focus on propositional programs.

All theorems hold for infinite programs.

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finite	$\longrightarrow$	$\ddot{\frown}$
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Syntax of logic programs

What is a logic program?

It is *a set of rules*:

$$\mathtt{a} \leftarrow \mathtt{b}_1 \;,\; \ldots \;,\; \mathtt{b}_n \qquad \text{(LP)}$$

$$\mathtt{a}_1 \vee \cdots \vee \mathtt{a}_k \leftarrow \mathtt{b}_1 \;,\; \ldots \;,\; \mathtt{b}_n \qquad \text{(DLP)}$$

$$\mathtt{a} \leftarrow \mathtt{b}_1 \;,\; \ldots \;,\; \mathtt{b}_n \;,\; \sim\mathtt{c}_1 \;,\; \ldots \;,\; \sim\mathtt{c}_m \qquad \text{(LPN)}$$

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Definition (Four programming languages)

A **DLPN program** is a set of DLPN-rules.

A rule without a head (**goal**) represents a **query** to the system:

$$\leftarrow \mathtt{p} \qquad \text{LP(N)}$$

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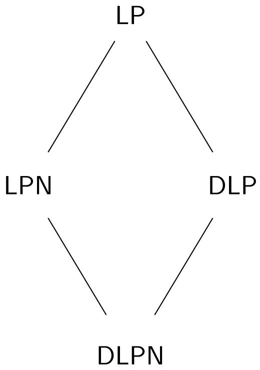

Model-theoretic semantics

LP: least Herbrand model

(van Emden & Kowalski, 1976)

LPN: well-founded model

(Van Gelder et al., 1991)

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DLP: minimal models

(Minker, 1982)

DLPN: ∞-valued minimal models

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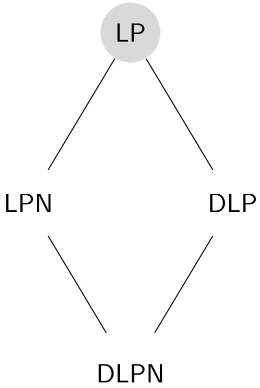

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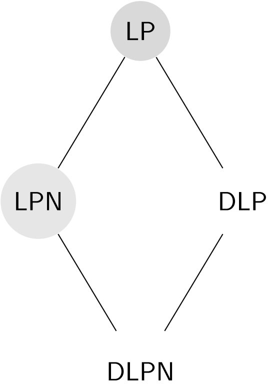

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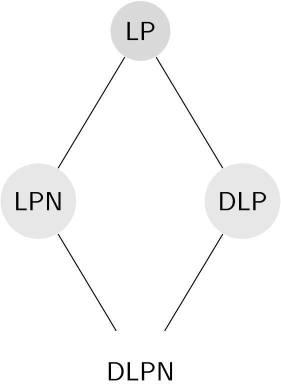

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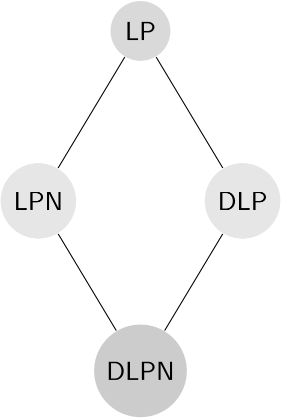

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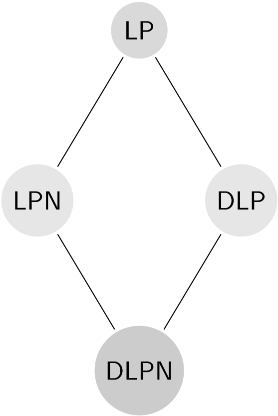

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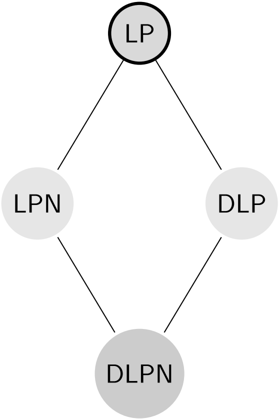

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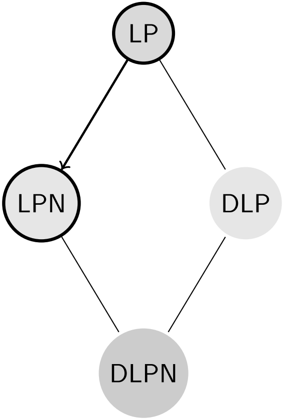

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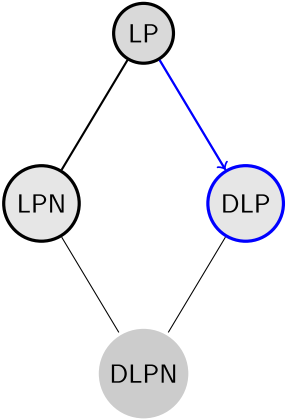

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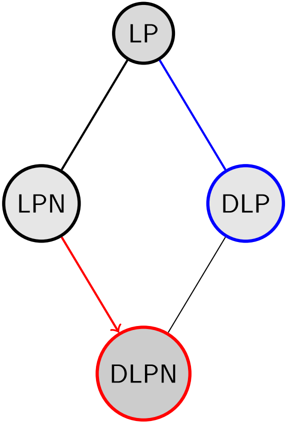

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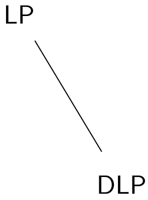

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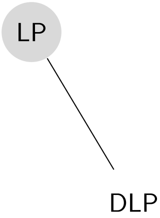

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Herbrand base, interpretations, and models through an example

$$\mathcal{P} := \left\{ \begin{array}{l} \texttt{p} \leftarrow \texttt{a} \;,\; \texttt{b} \\ \texttt{p} \leftarrow \texttt{c} \\ \texttt{a} \leftarrow \texttt{e} \\ \texttt{b} \leftarrow \texttt{d} \\ \texttt{b} \leftarrow \texttt{e} \\ \texttt{e} \leftarrow \\ \texttt{f} \leftarrow \end{array} \right\}$$

- At least one Herbrand model exists (the Herbrand Base)
- Model intersection property (mip)

$\therefore$ Existence of a $\subseteq$-least Herbrand model (LHM).
This must be the model the programmer had in mind:
it provides the semantics.

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Herbrand base $\{\mathtt{p}, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}, \mathtt{e}, \mathtt{f}\}$

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$$\mathcal{P} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{a}\ ,\ \mathtt{b} \\ \mathtt{p} \leftarrow \mathtt{c} \\ \mathtt{a} \leftarrow \mathtt{e} \\ \mathtt{b} \leftarrow \mathtt{d} \\ \mathtt{b} \leftarrow \mathtt{e} \\ \mathtt{e} \leftarrow \\ \mathtt{f} \leftarrow \end{array} \right\}$$

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- At least one Herbrand model exists (the Herbrand Base)
- Model intersection property (mip)

$\therefore$ Existence of a $\subseteq$-least Herbrand model (LHM).
This must be the model the programmer had in mind:
it provides the semantics.

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Fixpoint construction of the Least Herbrand Model

$$\mathcal{P} := \left\{ \begin{array}{l} \texttt{p} \leftarrow \texttt{a , b} \\ \texttt{p} \leftarrow \texttt{c} \\ \texttt{a} \leftarrow \texttt{e} \\ \texttt{b} \leftarrow \texttt{d} \\ \texttt{b} \leftarrow \texttt{e} \\ \texttt{e} \leftarrow \\ \texttt{f} \leftarrow \end{array} \right\}$$

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$\{\mathtt{e}, \mathtt{f}\}$

$\{\mathtt{e}, \mathtt{f}, \mathtt{a}, \mathtt{b}\}$

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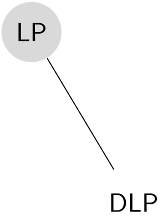
Least Herbrand Model of $\mathcal{P} = \{\mathtt{e}, \mathtt{f}, \mathtt{a}, \mathtt{b}, \mathtt{p}\}$

DLP

Model-theoretic semantics



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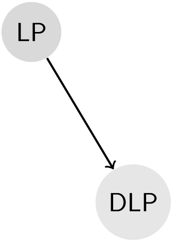

DLP

An example

Example

Consider the disjunctive program

$$\mathcal{P} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{a} \vee \mathtt{b} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}.$$

It has three models:

$$\{\mathtt{a}, \mathtt{p}\}, \quad \{\mathtt{b}, \mathtt{p}\}, \quad \{\mathtt{a}, \mathtt{b}, \mathtt{p}\},$$

none of which is least! (However, the first two are *minimal*.)
Notice that the intersection of all its models is $\{\mathtt{p}\}$, which is *not* a model.

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DLP

Model-theoretic semantics

Difficulties

- We no longer have a least model. ☹
- We have a set of minimal models—but there's no mip!

Approach (Minker 1982)

- We can consider this set to be the meaning of our program.
- The goal $\leftarrow$ `G` succeeds if `G` is **T** in every minimal model of $\mathcal{P}$.

DLP

Model-theoretic semantics

Example

Consider the DLP program

$$\mathcal{Q} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{b} \leftarrow \mathtt{c} \\ \mathtt{a} \lor \mathtt{c} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}.$$

Look at its models: $\{\mathrm{a}, \mathrm{p}\}$, $\{\mathrm{a}, \mathrm{b}, \mathrm{p}\}$, $\{\mathrm{c}, \mathrm{b}, \mathrm{p}\}$, $\{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{p}\}$.
E.g., $\mathrm{a} \lor \mathrm{b}$ is **T**, while a is **F**.

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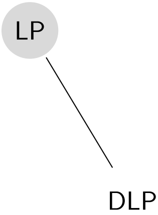
Game semantics

Why games?

- Nice denotational semantics for LP.
- We want to add more features in our language ($\sim$ and $\vee$).
- It becomes difficult to deal with them in a uniform way.
- Instead we look at **games**.

LP

LP game semantics



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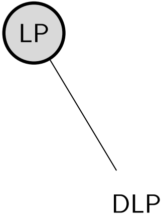

The LP game

The idea

Given a program $\mathcal{P}$ and a goal clause $\leftarrow \mathsf{q}$, a game will determine the goal's success and therefore the truth value of q.

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Layout of the games...

- Two players (Doubter vs Believer, Opponent vs Player):
  - Doubter who doubts "things" from bodies of rules;
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- A player who can't make a legal move, loses.
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- A player who can't make a legal move, loses.
- Doubter has the "*benefit of the doubt*".

The LP game

Example plays (1)

$$\mathcal{P} := \left\{ \begin{array}{l} \texttt{p} \leftarrow \texttt{a}\ ,\ \texttt{b} \\ \texttt{p} \leftarrow \texttt{c} \\ \texttt{a} \leftarrow \texttt{e} \\ \texttt{b} \leftarrow \texttt{d} \\ \texttt{b} \leftarrow \texttt{e} \\ \texttt{e} \leftarrow \\ \texttt{f} \leftarrow \end{array} \right\}$$

goal : $\leftarrow \texttt{p}$

The LP game

Example plays (1)

$$\mathcal{P} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{a}\ ,\ \mathtt{b} \\ \mathtt{p} \leftarrow \mathtt{c} \\ \mathtt{a} \leftarrow \mathtt{e} \\ \mathtt{b} \leftarrow \mathtt{d} \\ \mathtt{b} \leftarrow \mathtt{e} \\ \mathtt{e} \leftarrow \\ \mathtt{f} \leftarrow \end{array} \right\}$$

goal : $\leftarrow \mathtt{p}$

The LP game

Example plays (1)

$$\mathcal{P} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{a}\ ,\ \mathtt{b} \\ \mathtt{p} \leftarrow \mathtt{c} \\ \mathtt{a} \leftarrow \mathtt{e} \\ \mathtt{b} \leftarrow \mathtt{d} \\ \mathtt{b} \leftarrow \mathtt{e} \\ \mathtt{e} \leftarrow \\ \mathtt{f} \leftarrow \end{array} \right\}$$

goal : $\leftarrow \underline{\mathtt{p}}$

The LP game

Example plays (1)

$$\mathcal{P} := \left\{ \begin{array}{l} \color{blue}{\texttt{p} \leftarrow \texttt{a} \; \texttt{,} \; \texttt{b}} \\ \color{blue}{\texttt{p} \leftarrow \texttt{c}} \\ \texttt{a} \leftarrow \texttt{e} \\ \texttt{b} \leftarrow \texttt{d} \\ \texttt{b} \leftarrow \texttt{e} \\ \texttt{e} \leftarrow \\ \texttt{f} \leftarrow \end{array} \right\}$$

goal : $\leftarrow \underline{\texttt{p}}$

B_0 :

The LP game

Example plays (1)

$$\mathcal{P} := \left\{ \begin{array}{l} \texttt{p} \leftarrow \texttt{a , b} \\ \texttt{p} \leftarrow \texttt{c} \\ \texttt{a} \leftarrow \texttt{e} \\ \texttt{b} \leftarrow \texttt{d} \\ \texttt{b} \leftarrow \texttt{e} \\ \texttt{e} \leftarrow \\ \texttt{f} \leftarrow \end{array} \right\}$$

$$\left| \begin{array}{rl} \text{goal}: & \leftarrow \underline{\texttt{p}} \\ B_0: & \texttt{p} \leftarrow \texttt{a , b} \\ & \end{array} \right|$$

The LP game

Example plays (1)

$$\mathcal{P} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{a}\ ,\ \mathtt{b} \\ \mathtt{p} \leftarrow \mathtt{c} \\ \mathtt{a} \leftarrow \mathtt{e} \\ \mathtt{b} \leftarrow \mathtt{d} \\ \mathtt{b} \leftarrow \mathtt{e} \\ \mathtt{e} \leftarrow \\ \mathtt{f} \leftarrow \end{array} \right\}$$

$$\left| \begin{array}{rr} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ \mathsf{B}_0: & \mathtt{p} \leftarrow \color{red}{\mathtt{a}}\ ,\ \color{red}{\mathtt{b}} \end{array} \right|$$

The LP game

Example plays (1)

$$\mathcal{P} := \left\{ \begin{array}{l} \texttt{p} \leftarrow \texttt{a , b} \\ \texttt{p} \leftarrow \texttt{c} \\ \texttt{a} \leftarrow \texttt{e} \\ \texttt{b} \leftarrow \texttt{d} \\ \texttt{b} \leftarrow \texttt{e} \\ \texttt{e} \leftarrow \\ \texttt{f} \leftarrow \end{array} \right\}$$

$$\left| \begin{array}{ll} \text{goal}: & \leftarrow \underline{\texttt{p}} \\ B_0: & \texttt{p} \leftarrow \texttt{a , } \underline{\texttt{b}} \\ & \end{array} \right|$$

The LP game

Example plays (1)

$$\mathcal{P} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{a}\ ,\ \mathtt{b} \\ \mathtt{p} \leftarrow \mathtt{c} \\ \mathtt{a} \leftarrow \mathtt{e} \\ \color{blue}{\mathtt{b} \leftarrow \mathtt{d}} \\ \color{blue}{\mathtt{b} \leftarrow \mathtt{e}} \\ \mathtt{e} \leftarrow \\ \mathtt{f} \leftarrow \end{array} \right\}$$

$$\left| \begin{array}{rl} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ B_0: & \mathtt{p} \leftarrow \mathtt{a}\ ,\ \underline{\mathtt{b}} \\ B_1: & \end{array} \right|$$

The LP game

Example plays (1)

$$\mathcal{P} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{a}\ ,\ \mathtt{b} \\ \mathtt{p} \leftarrow \mathtt{c} \\ \mathtt{a} \leftarrow \mathtt{e} \\ \mathtt{b} \leftarrow \mathtt{d} \\ \mathtt{b} \leftarrow \mathtt{e} \\ \mathtt{e} \leftarrow \\ \mathtt{f} \leftarrow \end{array} \right\}$$

$$\left| \begin{array}{rl} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ B_0: & \mathtt{p} \leftarrow \mathtt{a}\ ,\ \underline{\mathtt{b}} \\ B_1: & \mathtt{b} \leftarrow \mathtt{d} \end{array} \right|$$

The LP game

Example plays (1)

$$\mathcal{P} := \left\{ \begin{array}{l} \texttt{p} \leftarrow \texttt{a} \ , \ \texttt{b} \\ \texttt{p} \leftarrow \texttt{c} \\ \texttt{a} \leftarrow \texttt{e} \\ \texttt{b} \leftarrow \texttt{d} \\ \texttt{b} \leftarrow \texttt{e} \\ \texttt{e} \leftarrow \\ \texttt{f} \leftarrow \end{array} \right\}$$

$$\left| \begin{array}{rl} \text{goal}: & \leftarrow \underline{\texttt{p}} \\ B_0 : & \texttt{p} \leftarrow \texttt{a} \ , \ \underline{\texttt{b}} \\ B_1 : & \texttt{b} \leftarrow \texttt{d} \end{array} \right|$$

The LP game

Example plays (1)

$$\mathcal{P} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{a}\ ,\ \mathtt{b} \\ \mathtt{p} \leftarrow \mathtt{c} \\ \mathtt{a} \leftarrow \mathtt{e} \\ \mathtt{b} \leftarrow \mathtt{d} \\ \mathtt{b} \leftarrow \mathtt{e} \\ \mathtt{e} \leftarrow \\ \mathtt{f} \leftarrow \end{array} \right\}$$

$$\left| \begin{array}{ll} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ B_0: & \mathtt{p} \leftarrow \mathtt{a}\ ,\ \underline{\mathtt{b}} \\ B_1: & \mathtt{b} \leftarrow \underline{\mathtt{d}} \end{array} \right|$$

The LP game

Example plays (1)

$$\mathcal{P} := \left\{ \begin{array}{l} \texttt{p} \leftarrow \texttt{a} \texttt{ , } \texttt{b} \\ \texttt{p} \leftarrow \texttt{c} \\ \texttt{a} \leftarrow \texttt{e} \\ \texttt{b} \leftarrow \texttt{d} \\ \texttt{b} \leftarrow \texttt{e} \\ \texttt{e} \leftarrow \\ \texttt{f} \leftarrow \end{array} \right\}$$

$$\left| \begin{array}{rl} \text{goal}: & \leftarrow \underline{\texttt{p}} \\ B_0: & \texttt{p} \leftarrow \texttt{a} \texttt{ , } \underline{\texttt{b}} \\ B_1: & \texttt{b} \leftarrow \underline{\texttt{d}} \end{array} \right|$$

Believer lost! ☹

The LP game

Example plays (2)

$$\mathcal{P} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{a}\ ,\ \mathtt{b} \\ \mathtt{p} \leftarrow \mathtt{c} \\ \mathtt{a} \leftarrow \mathtt{e} \\ \mathtt{b} \leftarrow \mathtt{d} \\ \mathtt{b} \leftarrow \mathtt{e} \\ \mathtt{e} \leftarrow \\ \mathtt{f} \leftarrow \end{array} \right\}$$

goal: $\leftarrow \mathtt{p}$

The LP game

Example plays (2)

$$\mathcal{P} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{a}\ ,\ \mathtt{b} \\ \mathtt{p} \leftarrow \mathtt{c} \\ \mathtt{a} \leftarrow \mathtt{e} \\ \mathtt{b} \leftarrow \mathtt{d} \\ \mathtt{b} \leftarrow \mathtt{e} \\ \mathtt{e} \leftarrow \\ \mathtt{f} \leftarrow \end{array} \right\}$$

goal : $\leftarrow \mathtt{p}$

The LP game

Example plays (2)

$$\mathcal{P} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{a}\ ,\ \mathtt{b} \\ \mathtt{p} \leftarrow \mathtt{c} \\ \mathtt{a} \leftarrow \mathtt{e} \\ \mathtt{b} \leftarrow \mathtt{d} \\ \mathtt{b} \leftarrow \mathtt{e} \\ \mathtt{e} \leftarrow \\ \mathtt{f} \leftarrow \end{array} \right\}$$

goal : $\leftarrow \underline{\mathtt{p}}$

The LP game

Example plays (2)

$$\mathcal{P} := \left\{ \begin{array}{l} \texttt{p} \leftarrow \texttt{a} \texttt{,} \texttt{b} \\ \texttt{p} \leftarrow \texttt{c} \\ \texttt{a} \leftarrow \texttt{e} \\ \texttt{b} \leftarrow \texttt{d} \\ \texttt{b} \leftarrow \texttt{e} \\ \texttt{e} \leftarrow \\ \texttt{f} \leftarrow \end{array} \right\}$$

goal :	$\leftarrow \underline{\texttt{p}}$
B_0 :	

The LP game

Example plays (2)

$$\mathcal{P} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{a}\ ,\ \mathtt{b} \\ \mathtt{p} \leftarrow \mathtt{c} \\ \mathtt{a} \leftarrow \mathtt{e} \\ \mathtt{b} \leftarrow \mathtt{d} \\ \mathtt{b} \leftarrow \mathtt{e} \\ \mathtt{e} \leftarrow \\ \mathtt{f} \leftarrow \end{array} \right\}$$

$$\left| \begin{array}{rl} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ B_0: & \mathtt{p} \leftarrow \mathtt{a}\ ,\ \mathtt{b} \end{array} \right|$$

The LP game

Example plays (2)

$$\mathcal{P} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{a}\ ,\ \mathtt{b} \\ \mathtt{p} \leftarrow \mathtt{c} \\ \mathtt{a} \leftarrow \mathtt{e} \\ \mathtt{b} \leftarrow \mathtt{d} \\ \mathtt{b} \leftarrow \mathtt{e} \\ \mathtt{e} \leftarrow \\ \mathtt{f} \leftarrow \end{array} \right\}$$

$$\left| \begin{array}{rr} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ B_0: & \mathtt{p} \leftarrow \color{red}{\mathtt{a}}\ ,\ \color{red}{\mathtt{b}} \end{array} \right|$$

The LP game

Example plays (2)

$$\mathcal{P} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{a} \ , \ \mathtt{b} \\ \mathtt{p} \leftarrow \mathtt{c} \\ \mathtt{a} \leftarrow \mathtt{e} \\ \mathtt{b} \leftarrow \mathtt{d} \\ \mathtt{b} \leftarrow \mathtt{e} \\ \mathtt{e} \leftarrow \\ \mathtt{f} \leftarrow \end{array} \right\}$$

$$\left| \begin{array}{rr} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ B_0: & \mathtt{p} \leftarrow \underline{\mathtt{a}} \ , \ \mathtt{b} \end{array} \right|$$

The LP game

Example plays (2)

$$\mathcal{P} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{a}\ ,\ \mathtt{b} \\ \mathtt{p} \leftarrow \mathtt{c} \\ \mathtt{a} \leftarrow \mathtt{e} \\ \mathtt{b} \leftarrow \mathtt{d} \\ \mathtt{b} \leftarrow \mathtt{e} \\ \mathtt{e} \leftarrow \\ \mathtt{f} \leftarrow \end{array} \right\}$$

$$\left| \begin{array}{ll} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ B_0: & \mathtt{p} \leftarrow \underline{\mathtt{a}}\ ,\ \mathtt{b} \\ B_1: & \end{array} \right|$$

The LP game

Example plays (2)

$$\mathcal{P} := \left\{ \begin{array}{l} \texttt{p} \leftarrow \texttt{a , b} \\ \texttt{p} \leftarrow \texttt{c} \\ \texttt{a} \leftarrow \texttt{e} \\ \texttt{b} \leftarrow \texttt{d} \\ \texttt{b} \leftarrow \texttt{e} \\ \texttt{e} \leftarrow \\ \texttt{f} \leftarrow \end{array} \right\}$$

$$\begin{array}{|rl|} \text{goal}: & \leftarrow \underline{\texttt{p}} \\ B_0: & \texttt{p} \leftarrow \underline{\texttt{a}}\texttt{ , b} \\ B_1: & \texttt{a} \leftarrow \texttt{e} \\ & \end{array}$$

The LP game

Example plays (2)

$$\mathcal{P} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{a}\ ,\ \mathtt{b} \\ \mathtt{p} \leftarrow \mathtt{c} \\ \mathtt{a} \leftarrow \mathtt{e} \\ \mathtt{b} \leftarrow \mathtt{d} \\ \mathtt{b} \leftarrow \mathtt{e} \\ \mathtt{e} \leftarrow \\ \mathtt{f} \leftarrow \end{array} \right\}$$

$$\begin{array}{rl} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ B_0: & \mathtt{p} \leftarrow \underline{\mathtt{a}}\ ,\ \mathtt{b} \\ B_1: & \mathtt{a} \leftarrow \color{red}{\mathtt{e}} \end{array}$$

The LP game

Example plays (2)

$$\mathcal{P} := \left\{ \begin{array}{l} \texttt{p} \leftarrow \texttt{a , b} \\ \texttt{p} \leftarrow \texttt{c} \\ \texttt{a} \leftarrow \texttt{e} \\ \texttt{b} \leftarrow \texttt{d} \\ \texttt{b} \leftarrow \texttt{e} \\ \texttt{e} \leftarrow \\ \texttt{f} \leftarrow \end{array} \right\}$$

$$\left| \begin{array}{rl} \text{goal}: & \leftarrow \underline{\texttt{p}} \\ B_0: & \texttt{p} \leftarrow \underline{\texttt{a}}\texttt{ , b} \\ B_1: & \texttt{a} \leftarrow \underline{\texttt{e}} \end{array} \right|$$

The LP game

Example plays (2)

$$\mathcal{P} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{a} \; , \; \mathtt{b} \\ \mathtt{p} \leftarrow \mathtt{c} \\ \mathtt{a} \leftarrow \mathtt{e} \\ \mathtt{b} \leftarrow \mathtt{d} \\ \mathtt{b} \leftarrow \mathtt{e} \\ \color{blue}{\mathtt{e} \leftarrow} \\ \mathtt{f} \leftarrow \end{array} \right\}$$

$$\left| \begin{array}{rl} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ B_0: & \mathtt{p} \leftarrow \underline{\mathtt{a}} \; , \; \mathtt{b} \\ B_1: & \mathtt{a} \leftarrow \underline{\mathtt{e}} \\ B_2: & \end{array} \right|$$

The LP game

Example plays (2)

$$\mathcal{P} := \left\{ \begin{array}{l} \texttt{p} \leftarrow \texttt{a , b} \\ \texttt{p} \leftarrow \texttt{c} \\ \texttt{a} \leftarrow \texttt{e} \\ \texttt{b} \leftarrow \texttt{d} \\ \texttt{b} \leftarrow \texttt{e} \\ \texttt{e} \leftarrow \\ \texttt{f} \leftarrow \end{array} \right\}$$

$$\left| \begin{array}{rl} \text{goal}: & \leftarrow \underline{\texttt{p}} \\ \text{B}_0: & \texttt{p} \leftarrow \underline{\texttt{a}}\texttt{ , b} \\ \text{B}_1: & \texttt{a} \leftarrow \underline{\texttt{e}} \\ \text{B}_2: & \texttt{e} \leftarrow \end{array} \right|$$

The LP game

Example plays (2)

$$\mathcal{P} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{a}\ ,\ \mathtt{b} \\ \mathtt{p} \leftarrow \mathtt{c} \\ \mathtt{a} \leftarrow \mathtt{e} \\ \mathtt{b} \leftarrow \mathtt{d} \\ \mathtt{b} \leftarrow \mathtt{e} \\ \mathtt{e} \leftarrow \\ \mathtt{f} \leftarrow \end{array} \right\}$$

$$\left| \begin{array}{rl} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ B_0: & \mathtt{p} \leftarrow \underline{\mathtt{a}}\ ,\ \mathtt{b} \\ B_1: & \mathtt{a} \leftarrow \underline{\mathtt{e}} \\ B_2: & \mathtt{e} \leftarrow \emptyset \end{array} \right|$$

The LP game

Example plays (2)

$$\mathcal{P} := \left\{ \begin{array}{l} \texttt{p} \leftarrow \texttt{a} \ , \ \texttt{b} \\ \texttt{p} \leftarrow \texttt{c} \\ \texttt{a} \leftarrow \texttt{e} \\ \texttt{b} \leftarrow \texttt{d} \\ \texttt{b} \leftarrow \texttt{e} \\ \texttt{e} \leftarrow \\ \texttt{f} \leftarrow \end{array} \right\}$$

$$\left| \begin{array}{rl} \text{goal}: & \leftarrow \underline{\texttt{p}} \\ B_0: & \texttt{p} \leftarrow \underline{\texttt{a}} \ , \ \texttt{b} \\ B_1: & \texttt{a} \leftarrow \underline{\texttt{e}} \\ B_2: & \texttt{e} \leftarrow \end{array} \right|$$

Believer wins! ☺

The LP game

Example plays (3)

$$\mathcal{Q} := \left\{ \begin{matrix} \mathtt{p} \leftarrow \mathtt{q} \\ \mathtt{q} \leftarrow \mathtt{p} \end{matrix} \right\}$$

goal :	$\leftarrow \mathtt{p}$

The LP game

Example plays (3)

$$\mathcal{Q} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{q} \\ \mathtt{q} \leftarrow \mathtt{p} \end{array} \right\}$$

goal :	$\leftarrow \mathtt{p}$

The LP game

Example plays (3)

$$\mathcal{Q} := \left\{ \begin{matrix} \mathtt{p} \leftarrow \mathtt{q} \\ \mathtt{q} \leftarrow \mathtt{p} \end{matrix} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$

The LP game

Example plays (3)

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	

$$\mathcal{Q} := \left\{ \begin{array}{l} \color{blue}{\mathtt{p} \leftarrow \mathtt{q}} \\ \mathtt{q} \leftarrow \mathtt{p} \end{array} \right\}$$

The LP game

Example plays (3)

$$\mathcal{Q} := \begin{Bmatrix} \mathtt{p} \leftarrow \mathtt{q} \\ \mathtt{q} \leftarrow \mathtt{p} \end{Bmatrix}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	$\mathtt{p} \leftarrow \mathtt{q}$

The LP game

Example plays (3)

$$\mathcal{Q} := \left\{ \begin{array}{l} \mathrm{p} \leftarrow \mathrm{q} \\ \mathrm{q} \leftarrow \mathrm{p} \end{array} \right\}$$

goal :	$\leftarrow \underline{\mathrm{p}}$
B_0 :	$\mathrm{p} \leftarrow \mathrm{q}$

The LP game

Example plays (3)

$$\mathcal{Q} := \left\{ \begin{matrix} \mathtt{p} \leftarrow \mathtt{q} \\ \mathtt{q} \leftarrow \mathtt{p} \end{matrix} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	$\mathtt{p} \leftarrow \underline{\mathtt{q}}$

The LP game

Example plays (3)

$$\mathcal{Q} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{q} \\ \mathtt{q} \leftarrow \mathtt{p} \end{array} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	$\mathtt{p} \leftarrow \underline{\mathtt{q}}$
B_1 :	

The LP game

Example plays (3)

$$\mathcal{Q} := \left\{ \begin{matrix} \mathtt{p} \leftarrow \mathtt{q} \\ \mathtt{q} \leftarrow \mathtt{p} \end{matrix} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	$\mathtt{p} \leftarrow \underline{\mathtt{q}}$
B_1 :	$\mathtt{q} \leftarrow \mathtt{p}$

The LP game

Example plays (3)

$$\mathcal{Q} := \left\{ \begin{matrix} \mathrm{p} \leftarrow \mathrm{q} \\ \mathrm{q} \leftarrow \mathrm{p} \end{matrix} \right\}$$

goal :	$\leftarrow \underline{\mathrm{p}}$
B_0 :	$\mathrm{p} \leftarrow \underline{\mathrm{q}}$
B_1 :	$\mathrm{q} \leftarrow \mathrm{p}$

The LP game

Example plays (3)

$$\mathcal{Q} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{q} \\ \mathtt{q} \leftarrow \mathtt{p} \end{array} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	$\mathtt{p} \leftarrow \underline{\mathtt{q}}$
B_1 :	$\mathtt{q} \leftarrow \underline{\mathtt{p}}$

The LP game

Example plays (3)

$$\mathcal{Q} := \left\{ \begin{array}{l} \color{blue}{p \leftarrow q} \\ q \leftarrow p \end{array} \right\}$$

goal :	$\leftarrow \underline{p}$
B_0 :	$p \leftarrow \underline{q}$
B_1 :	$q \leftarrow \underline{p}$
B_2 :	

The LP game

Example plays (3)

$$\mathcal{Q} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{q} \\ \mathtt{q} \leftarrow \mathtt{p} \end{array} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	$\mathtt{p} \leftarrow \underline{\mathtt{q}}$
B_1 :	$\mathtt{q} \leftarrow \underline{\mathtt{p}}$
B_2 :	$\mathtt{p} \leftarrow \mathtt{q}$

The LP game

Example plays (3)

$$\mathcal{Q} := \left\{ \begin{matrix} \mathtt{p} \leftarrow \mathtt{q} \\ \mathtt{q} \leftarrow \mathtt{p} \end{matrix} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	$\mathtt{p} \leftarrow \underline{\mathtt{q}}$
B_1 :	$\mathtt{q} \leftarrow \underline{\mathtt{p}}$
B_2 :	$\mathtt{p} \leftarrow \underline{\mathtt{q}}$

The LP game

Example plays (3)

$$\mathcal{Q} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{q} \\ \mathtt{q} \leftarrow \mathtt{p} \end{array} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	$\mathtt{p} \leftarrow \underline{\mathtt{q}}$
B_1 :	$\mathtt{q} \leftarrow \underline{\mathtt{p}}$
B_2 :	$\mathtt{p} \leftarrow \underline{\mathtt{q}}$
B_3 :	$\mathtt{q} \leftarrow \underline{\mathtt{p}}$
B_4 :	$\mathtt{p} \leftarrow \underline{\mathtt{q}}$
$\vdots$	$\vdots$

The LP game

Example plays (3)

$$\mathcal{Q} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \mathtt{q} \\ \mathtt{q} \leftarrow \mathtt{p} \end{array} \right\}$$

$$\left| \begin{array}{rr} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ \mathsf{B}_0: & \mathtt{p} \leftarrow \underline{\mathtt{q}} \\ \mathsf{B}_1: & \mathtt{q} \leftarrow \underline{\mathtt{p}} \\ \mathsf{B}_2: & \mathtt{p} \leftarrow \underline{\mathtt{q}} \\ \mathsf{B}_3: & \mathtt{q} \leftarrow \underline{\mathtt{p}} \\ \mathsf{B}_4: & \mathtt{p} \leftarrow \underline{\mathtt{q}} \\ \vdots & \vdots \end{array} \right|$$

(benefit of the doubt) $\Longrightarrow$ **Believer lost!** ☹

The LP game

How to get semantics out of it

We look at strategies

The value of p w.r.t. $\mathcal{P}$ is determined by the game:
$\leftarrow$ p succeeds iff there is a winning strategy (for the Believer).

The connecting result

Theorem (Di Cosmo, Loddo & Nicolet)

$$\textbf{LPG} \approx \textbf{LHM}$$

The LP game

How to get semantics out of it

We look at strategies

The value of p w.r.t. $\mathcal{P}$ is determined by the game:
$\leftarrow$ p succeeds iff there is a winning strategy (for the Believer).

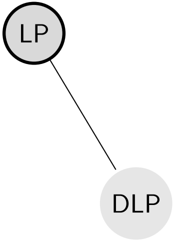
The connecting result

Theorem (Di Cosmo, Loddo & Nicolet)

$$\textbf{LPG} \approx \textbf{LHM}$$

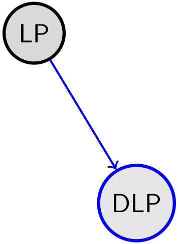
DLP

DLP game semantics



DLP

DLP game semantics



From LP to DLP (1)

Let's try the LP game

$$\mathcal{R} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \\ \mathtt{q} \leftarrow \mathtt{a} \end{array} \right\} \quad \Big| \text{goal}: \quad \leftarrow \mathtt{p} \vee \mathtt{q} \quad \Big|$$

From LP to DLP (1)

Let's try the LP game

$$\mathcal{R} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \\ \mathtt{q} \leftarrow \mathtt{a} \end{array} \right\} \quad \Big| \text{goal} : \quad \leftarrow \mathtt{p} \vee \mathtt{q} \quad \Big|$$

From LP to DLP (1)

Let's try the LP game

$$\mathcal{R} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \\ \mathtt{q} \leftarrow \mathtt{a} \end{array} \right\} \quad \Big| \text{goal} : \qquad \leftarrow \underline{\mathtt{p} \vee \mathtt{q}} \qquad \Big|$$

From LP to DLP (1)

Let's try the LP game

$$\mathcal{R} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \\ \mathtt{q} \leftarrow \mathtt{a} \end{array} \right\} \quad \Big| \text{goal} : \quad \leftarrow \underline{\mathtt{p} \vee \mathtt{q}} \quad \Big|$$

Believer lost! $\ddot{\frown}$

From LP to DLP (1)

(1) Believer justifies subsets

Believer can justify any non-empty subset of the doubted disjunction.

From LP to DLP (1)

LP game + subset

$$\mathcal{R} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \\ \mathtt{q} \leftarrow \mathtt{a} \end{array} \right\}$$

goal : $\leftarrow \underline{\mathtt{p} \vee \mathtt{q}}$

From LP to DLP (1)

LP game + subset

$$\mathcal{R} := \left\{ \begin{array}{l} p \leftarrow \\ q \leftarrow a \end{array} \right\} \quad \begin{array}{|ll|} \text{goal :} & \leftarrow \underline{p \vee q} \\ B_0 : & \end{array}$$

From LP to DLP (1)

LP game + subset

$$\mathcal{R} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \\ \mathtt{q} \leftarrow \mathtt{a} \end{array} \right\} \quad \left| \begin{array}{ll} \text{goal}: & \leftarrow \underline{\mathtt{p} \vee \mathtt{q}} \\ \mathsf{B}_0: & \mathtt{p} \leftarrow \end{array} \right|$$

From LP to DLP (1)

LP game + subset

$$\mathcal{R} := \left\{ \begin{array}{l} \mathtt{p} \leftarrow \\ \mathtt{q} \leftarrow \mathtt{a} \end{array} \right\} \quad \left| \begin{array}{rr} \text{goal :} & \leftarrow \underline{\mathtt{p} \vee \mathtt{q}} \\ \mathsf{B}_0 : & \mathtt{p} \leftarrow \emptyset \end{array} \right|$$

Believer wins! ⌣̈

From LP to DLP (2)

LP game + subset

$$\mathcal{P} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{a} \vee \mathtt{b} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}$$

goal : $\leftarrow \mathtt{p}$

From LP to DLP (2)

LP game + subset

$$\mathcal{P} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{a} \vee \mathtt{b} \leftarrow \end{array} \right\}$$

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goal : $\leftarrow \underline{\mathtt{p}}$

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goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	

From LP to DLP (2)

LP game + subset

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From LP to DLP (2)

LP game + subset

$$\mathcal{P} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ a \vee b \leftarrow \end{array} \right\}$$

goal :	$\leftarrow \underline{p}$
B_0 :	$p \leftarrow \underline{a}$

From LP to DLP (2)

LP game + subset

$$\mathcal{P} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{a} \vee \mathtt{b} \leftarrow \phantom{\mathtt{b}} \end{array} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	$\mathtt{p} \leftarrow \mathtt{a}$
B_1 :	☹

From LP to DLP (2)

LP game + subset

$$\mathcal{P} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{a} \vee \mathtt{b} \leftarrow \end{array} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	$\mathtt{p} \leftarrow \mathtt{b}$

From LP to DLP (2)

LP game + subset

$$\mathcal{P} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{a} \vee \mathtt{b} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	$\mathtt{p} \leftarrow \underline{\mathtt{b}}$

From LP to DLP (2)

LP game + subset

$$\mathcal{P} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{a} \vee \mathtt{b} \leftarrow \quad \end{array} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	$\mathtt{p} \leftarrow \mathtt{b}$
B_1 :	⌢̈

From LP to DLP (2)

(2) Combining rules

Believer is allowed to use more than one rule (**combo** move).

Meaning of combination

Notice, that if we accept the two rules

$$H \leftarrow B \qquad \text{and} \qquad H' \leftarrow B'$$

then their *combination*

$$H \vee H' \leftarrow B \vee B'$$

should also be accepted.

From LP to DLP (2)

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From LP to DLP (2)

LP game + subset + combo

$$\mathcal{P} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{a} \vee \mathtt{b} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}$$

goal : $\leftarrow \underline{\mathtt{p}}$

From LP to DLP (2)

LP game + subset + combo

$$\mathcal{P} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{a} \vee \mathtt{b} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	

From LP to DLP (2)

LP game + subset + combo

$$\mathcal{P} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{a} \vee \mathtt{b} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	$(\mathtt{p} \leftarrow \mathtt{a})$
	$(\mathtt{p} \leftarrow \mathtt{b})$

From LP to DLP (2)

LP game + subset + combo

$$\mathcal{P} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{a} \lor \mathtt{b} \leftarrow \end{array} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	$(\mathtt{p} \leftarrow \mathtt{a})$
	$(\mathtt{p} \leftarrow \mathtt{b})$
	$\mathtt{p} \leftarrow \mathtt{a} \lor \mathtt{b}$

From LP to DLP (2)

LP game + subset + combo

$$\mathcal{P} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{a} \vee \mathtt{b} \leftarrow \end{array} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	$(\mathtt{p} \leftarrow \mathtt{a})$
	$(\mathtt{p} \leftarrow \mathtt{b})$
	$\mathtt{p} \leftarrow \mathtt{a} \vee \mathtt{b}$

From LP to DLP (2)

LP game + subset + combo

$$\mathcal{P} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{a} \vee \mathtt{b} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}$$

$$\begin{array}{|rc|} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ B_0: & (\mathtt{p} \leftarrow \mathtt{a}) \\ & (\mathtt{p} \leftarrow \mathtt{b}) \\ & \mathtt{p} \leftarrow \underline{\mathtt{a} \vee \mathtt{b}} \\ B_1: & \end{array}$$

From LP to DLP (2)

LP game + subset + combo

$$\mathcal{P} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{a} \vee \mathtt{b} \leftarrow \end{array} \right\}$$

$$\begin{array}{|rr|} \text{goal}: & \leftarrow \underline{\mathtt{p}} \\ B_0: & (\mathtt{p} \leftarrow \mathtt{a}) \\ & (\mathtt{p} \leftarrow \mathtt{b}) \\ & \mathtt{p} \leftarrow \underline{\mathtt{a} \vee \mathtt{b}} \\ B_1: & \mathtt{a} \vee \mathtt{b} \leftarrow \end{array}$$

From LP to DLP (2)

LP game + subset + combo

$$\mathcal{P} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ a \vee b \leftarrow \end{array} \right\}$$

$$\left| \begin{array}{lr} \text{goal}: & \leftarrow \underline{p} \\ B_0: & (p \leftarrow a) \\ & (p \leftarrow b) \\ & p \leftarrow \underline{a \vee b} \\ B_1: & a \vee b \leftarrow \emptyset \end{array} \right|$$

Believer wins! ⌣̈

From LP to DLP (3)

LP game + subset + combo

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

goal :	$\leftarrow p$

From LP to DLP (3)

LP game + subset + combo

$$\mathcal{Q} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{b} \leftarrow \mathtt{c} \\ \mathtt{a} \vee \mathtt{c} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}$$

goal : $\leftarrow \mathtt{p}$

From LP to DLP (3)

LP game + subset + combo

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

goal :	$\leftarrow \underline{p}$

From LP to DLP (3)

LP game + subset + combo

$$\mathcal{Q} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{b} \leftarrow \mathtt{c} \\ \mathtt{a} \vee \mathtt{c} \leftarrow \phantom{\mathtt{c}} \end{array} \right\}$$

goal : $\leftarrow \underline{\mathtt{p}}$

From LP to DLP (3)

LP game + subset + combo

$$\mathcal{Q} := \left\{ \begin{aligned} p &\leftarrow a \\ p &\leftarrow b \\ b &\leftarrow c \\ a \lor c &\leftarrow \end{aligned} \right\}$$

goal :	$\leftarrow \underline{p}$
B_0 :	$(p \leftarrow a)$
	$(p \leftarrow b)$

From LP to DLP (3)

LP game + subset + combo

$$\mathcal{Q} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{b} \leftarrow \mathtt{c} \\ \mathtt{a} \vee \mathtt{c} \leftarrow \phantom{\mathtt{c}} \end{array} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	$(\mathtt{p} \leftarrow \mathtt{a})$
	$(\mathtt{p} \leftarrow \mathtt{b})$
	$\mathtt{p} \leftarrow \mathtt{a} \vee \mathtt{b}$

From LP to DLP (3)

LP game + subset + combo

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

goal :	$\leftarrow \underline{p}$
B_0 :	$(p \leftarrow a)$
	$(p \leftarrow b)$
	$p \leftarrow a \vee b$

From LP to DLP (3)

LP game + subset + combo

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \lor c \leftarrow \quad \end{array} \right\}$$

goal :	$\leftarrow \underline{p}$
B_0 :	$(p \leftarrow a)$
	$(p \leftarrow b)$
	$p \leftarrow \underline{a \lor b}$

From LP to DLP (3)

LP game + subset + combo

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

goal :	$\leftarrow \underline{p}$
B_0 :	$(p \leftarrow a)$
	$(p \leftarrow b)$
	$p \leftarrow \underline{a \vee b}$
B_1 :	$b \leftarrow c$

From LP to DLP (3)

LP game + subset + combo

$$\mathcal{Q} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{b} \leftarrow \mathtt{c} \\ \mathtt{a} \vee \mathtt{c} \leftarrow \phantom{\mathtt{c}} \end{array} \right\}$$

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B_1 :	$\mathtt{b} \leftarrow \mathtt{c}$

From LP to DLP (3)

LP game + subset + combo

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \lor c \leftarrow \end{array} \right\}$$

goal :	$\leftarrow \underline{p}$
B_0 :	$(p \leftarrow a)$
	$(p \leftarrow b)$
	$p \leftarrow \underline{a \lor b}$
B_1 :	$b \leftarrow \underline{c}$

From LP to DLP (3)

LP game + subset + combo

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

goal :	$\leftarrow \underline{p}$
B_0 :	$(p \leftarrow a)$
	$(p \leftarrow b)$
	$p \leftarrow \underline{a \vee b}$
B_1 :	$b \leftarrow \underline{c}$

Believer lost! $\ddot{\frown}$

From LP to DLP (3)

(3) Implicit rules

Believer is allowed to use *implicit* rules of the form

$$\mathtt{a} \leftarrow \mathtt{a},$$

not from the program.

From LP to DLP (3)

LP game + subset + combo + implicit

$$\{a \leftarrow a, \quad b \leftarrow b, \quad c \leftarrow c, \quad p \leftarrow p\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

goal : $\leftarrow \underline{p}$

From LP to DLP (3)

LP game + subset + combo + implicit

$$\{a \leftarrow a, \quad b \leftarrow b, \quad c \leftarrow c, \quad p \leftarrow p\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

goal : $\leftarrow \underline{p}$

B_0 :

From LP to DLP (3)

LP game + subset + combo + implicit

$$\{a \leftarrow a, \quad b \leftarrow b, \quad c \leftarrow c, \quad p \leftarrow p\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \quad \end{array} \right\}$$

goal :	$\leftarrow \underline{p}$
B_0 :	$p \leftarrow a \vee b$

From LP to DLP (3)

LP game + subset + combo + implicit

$$\{a \leftarrow a, \quad b \leftarrow b, \quad c \leftarrow c, \quad p \leftarrow p\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

goal :	$\leftarrow \underline{p}$
B_0 :	$p \leftarrow a \vee b$

From LP to DLP (3)

LP game + subset + combo + implicit

$$\{a \leftarrow a, \quad b \leftarrow b, \quad c \leftarrow c, \quad p \leftarrow p\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

goal :	$\leftarrow \underline{p}$
B_0 :	$p \leftarrow \underline{a \vee b}$

From LP to DLP (3)

LP game + subset + combo + implicit

$\{a \leftarrow a, \quad b \leftarrow b, \quad c \leftarrow c, \quad p \leftarrow p\}$

$$\mathcal{Q} \coloneqq \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

goal :	$\leftarrow \underline{p}$
B_0 :	$p \leftarrow \underline{a \vee b}$
B_1 :	

From LP to DLP (3)

LP game + subset + combo + implicit

$$\{a \leftarrow a, \quad b \leftarrow b, \quad c \leftarrow c, \quad p \leftarrow p\}$$

$$\mathcal{Q} \coloneqq \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

goal :	$\leftarrow \underline{p}$
B_0 :	$p \leftarrow \underline{a \vee b}$
B_1 :	$a \vee b \leftarrow a \vee c$

From LP to DLP (3)

LP game + subset + combo + implicit

$$\{a \leftarrow a, \quad b \leftarrow b, \quad c \leftarrow c, \quad p \leftarrow p\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

goal :	$\leftarrow \underline{p}$
B_0 :	$p \leftarrow \underline{a \vee b}$
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From LP to DLP (3)

LP game + subset + combo + implicit

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goal :	$\leftarrow \underline{p}$
B_0 :	$p \leftarrow \underline{a \vee b}$
B_1 :	$a \vee b \leftarrow \underline{a \vee c}$

From LP to DLP (3)

LP game + subset + combo + implicit

$$\{a \leftarrow a,\quad b \leftarrow b,\quad c \leftarrow c,\quad p \leftarrow p\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \lor c \leftarrow \end{array} \right\} \quad \left| \begin{array}{rr} \text{goal}: & \leftarrow \underline{p} \\ B_0: & p \leftarrow \underline{a \lor b} \\ B_1: & a \lor b \leftarrow \underline{a \lor c} \\ B_2: & \end{array} \right|$$

From LP to DLP (3)

LP game + subset + combo + implicit

$$\{a \leftarrow a, \quad b \leftarrow b, \quad c \leftarrow c, \quad p \leftarrow p\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ \color{blue}{a \lor c \leftarrow} \end{array} \right\} \quad \left| \begin{array}{ll} \text{goal}: & \leftarrow \underline{p} \\ B_0: & p \leftarrow \underline{a \lor b} \\ B_1: & a \lor b \leftarrow \underline{a \lor c} \\ B_2: & a \lor c \leftarrow \end{array} \right|$$

From LP to DLP (3)

LP game + subset + combo + implicit

$$\{a \leftarrow a, \quad b \leftarrow b, \quad c \leftarrow c, \quad p \leftarrow p\}$$

$$\mathcal{Q} \coloneqq \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\} \quad \begin{array}{|rr|} \text{goal}: & \leftarrow \underline{p} \\ B_0: & p \leftarrow \underline{a \vee b} \\ B_1: & a \vee b \leftarrow \underline{a \vee c} \\ B_2: & a \vee c \leftarrow \emptyset \end{array}$$

Believer wins! ⌣̈

The DLP game

Remark: what do we really need?

$$\text{DLP game} \stackrel{?}{=} \text{LP game} + \text{ subset } + \text{combo} + \text{implicit}$$

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We can obtain the “subset” by “combo” and “implicit”.

The DLP game

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The DLP game

Remark: what do we really need?

$$\text{DLP game} \stackrel{?}{=} \text{LP game} + \text{combo} + \text{implicit}$$

The DLP game

Finally...

DLP game = LP game + combo + implicit

The DLPG game

an example of stalling

$$\{\mathtt{a} \leftarrow \mathtt{a}, \quad \mathtt{b} \leftarrow \mathtt{b}, \quad \mathtt{c} \leftarrow \mathtt{c}, \quad \mathtt{p} \leftarrow \mathtt{p}\}$$

$$\mathcal{Q} \coloneqq \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{b} \leftarrow \mathtt{c} \\ \mathtt{a} \vee \mathtt{c} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}$$

goal : $\leftarrow \underline{\mathtt{p}}$

The DLPG game

an example of stalling

$$\{a \leftarrow a, \quad b \leftarrow b, \quad c \leftarrow c, \quad p \leftarrow p\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

goal : $\leftarrow \underline{p}$

B_0 :

The DLPG game

an example of stalling

$$\{\mathtt{a} \leftarrow \mathtt{a}, \quad \mathtt{b} \leftarrow \mathtt{b}, \quad \mathtt{c} \leftarrow \mathtt{c}, \quad \mathtt{p} \leftarrow \mathtt{p}\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{b} \leftarrow \mathtt{c} \\ \mathtt{a} \vee \mathtt{c} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	$\mathtt{p} \leftarrow \mathtt{a} \vee \mathtt{b}$

The DLPG game

an example of stalling

$$\{a \leftarrow a, \quad b \leftarrow b, \quad c \leftarrow c, \quad p \leftarrow p\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

goal :	$\leftarrow \underline{p}$
B_0 :	$p \leftarrow a \vee b$

The DLPG game

an example of stalling

$$\{a \leftarrow a, \quad b \leftarrow b, \quad c \leftarrow c, \quad p \leftarrow p\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \lor c \leftarrow \end{array} \right\}$$

goal :	$\leftarrow \underline{p}$
B_0 :	$p \leftarrow \underline{a \lor b}$

The DLPG game

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$$\{a \leftarrow a, \quad b \leftarrow b, \quad c \leftarrow c, \quad p \leftarrow p\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \quad \end{array} \right\}$$

goal :	$\leftarrow \underline{p}$
B_0 :	$p \leftarrow \underline{a \vee b}$
B_1 :	

The DLPG game

an example of stalling

$$\{\mathtt{a} \leftarrow \mathtt{a}, \quad \mathtt{b} \leftarrow \mathtt{b}, \quad \mathtt{c} \leftarrow \mathtt{c}, \quad \mathtt{p} \leftarrow \mathtt{p}\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{b} \leftarrow \mathtt{c} \\ \mathtt{a} \vee \mathtt{c} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	$\mathtt{p} \leftarrow \underline{\mathtt{a} \vee \mathtt{b}}$
B_1 :	$\mathtt{a} \vee \mathtt{b} \leftarrow \mathtt{a} \vee \mathtt{b}$

Believer is stalling. . .

The DLPG game

an example of stalling

$$\{a \leftarrow a, \quad b \leftarrow b, \quad c \leftarrow c, \quad p \leftarrow p\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

goal :	$\leftarrow \underline{p}$
B_0 :	$p \leftarrow \underline{a \vee b}$
B_1 :	$a \vee b \leftarrow a \vee b$

Believer is stalling...

The DLPG game

an example of stalling

$$\{a \leftarrow a, \quad b \leftarrow b, \quad c \leftarrow c, \quad p \leftarrow p\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

goal :	$\leftarrow \underline{p}$
B_0 :	$p \leftarrow \underline{a \vee b}$
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Believer is stalling...

The DLPG game

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$$\{a \leftarrow a, \quad b \leftarrow b, \quad c \leftarrow c, \quad p \leftarrow p\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

goal :	$\leftarrow \underline{p}$
B_0 :	$p \leftarrow \underline{a \vee b}$
B_1 :	$a \vee b \leftarrow \underline{a \vee b}$
B_2 :	

Believer is stalling...

The DLPG game

an example of stalling

$$\{\mathtt{a} \leftarrow \mathtt{a}, \quad \mathtt{b} \leftarrow \mathtt{b}, \quad \mathtt{c} \leftarrow \mathtt{c}, \quad \mathtt{p} \leftarrow \mathtt{p}\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{b} \leftarrow \mathtt{c} \\ \mathtt{a} \vee \mathtt{c} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}$$

goal :	$\leftarrow \underline{\mathtt{p}}$
B_0 :	$\mathtt{p} \leftarrow \underline{\mathtt{a} \vee \mathtt{b}}$
B_1 :	$\mathtt{a} \vee \mathtt{b} \leftarrow \underline{\mathtt{a} \vee \mathtt{b}}$
B_2 :	$\mathtt{a} \vee \mathtt{b} \leftarrow \mathtt{a} \vee \mathtt{b}$

Believer is stalling. . .

The DLPG game

an example of stalling

$$\{a \leftarrow a, \quad b \leftarrow b, \quad c \leftarrow c, \quad p \leftarrow p\}$$

$$\mathcal{Q} := \left\{ \begin{array}{r} p \leftarrow a \\ p \leftarrow b \\ b \leftarrow c \\ a \vee c \leftarrow \end{array} \right\}$$

goal :	$\leftarrow \underline{p}$
B_0 :	$p \leftarrow \underline{a \vee b}$
B_1 :	$a \vee b \leftarrow \underline{a \vee b}$
B_2 :	$a \vee b \leftarrow a \vee b$

Believer is stalling...

The DLPG game

an example of stalling

$$\{\mathtt{a} \leftarrow \mathtt{a}, \quad \mathtt{b} \leftarrow \mathtt{b}, \quad \mathtt{c} \leftarrow \mathtt{c}, \quad \mathtt{p} \leftarrow \mathtt{p}\}$$

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B_0 :	$\mathtt{p} \leftarrow \underline{\mathtt{a} \vee \mathtt{b}}$
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B_3 :	

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The DLPG game

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The DLPG game

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B_4 :	

The DLPG game

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$$\{\mathtt{a} \leftarrow \mathtt{a},\quad \mathtt{b} \leftarrow \mathtt{b},\quad \mathtt{c} \leftarrow \mathtt{c},\quad \mathtt{p} \leftarrow \mathtt{p}\}$$

$$\mathcal{Q} \coloneqq \left\{ \begin{array}{r} \mathtt{p} \leftarrow \mathtt{a} \\ \mathtt{p} \leftarrow \mathtt{b} \\ \mathtt{b} \leftarrow \mathtt{c} \\ \mathtt{a} \vee \mathtt{c} \leftarrow \phantom{\mathtt{a}} \end{array} \right\}
\qquad
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B_2 :	$a \vee b \leftarrow \underline{a \vee b}$
B_3 :	$a \vee b \leftarrow \underline{a \vee c}$
B_4 :	$a \vee c \leftarrow \emptyset$

The DLPG game

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Believer wins! ⌣̈

The DLP game

How to get semantics out of it

We look at strategies (The same as in LP!)

The value of $\mathrm{p}_1 \vee \cdots \vee \mathrm{p}_k$ w.r.t. $\mathcal{P}$ is determined by the game: $\leftarrow \mathrm{p}_1 \vee \cdots \vee \mathrm{p}_k$ succeeds iff there is a winning strategy (for the Believer).

The connecting result

Theorem (me, 2013)

DLPG $\approx$ MM

The DLP game

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The connecting result

Theorem (me, 2013)

$$\mathbf{DLPG} \approx \mathbf{MM}$$

Soundness and completeness

A proof sketch

We are given a DLP program $\mathcal{P}$ and a goal G.

Proof is by induction on the number of $\vee$ in heads of $\mathcal{P}$.

- We select a **disjunctive** rule ϕ from $\mathcal{P}$, e.g.,

$$\underbrace{\mathtt{p} \vee \mathtt{q} \vee \mathtt{r} \leftarrow \mathtt{a}\ ,\ \mathtt{b} \vee \mathtt{c}\ ,\ \mathtt{d}}_{\phi}$$

- and we **split it in two** rules, ϕ_1 and ϕ_2 by splitting its head:

- then we look at the *"less disjunctive"* programs $\mathcal{P}_1$ and $\mathcal{P}_2$ obtained by **replacing** ϕ by ϕ_1 and ϕ_2 respectively.

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Soundness and completeness

A proof sketch (cont'd)

Completeness (Minker semantics $\implies$ DLP game semantics)

- G is true in every minimal model of $\mathcal{P}$.
- We argue that it must be true in every minimal model of $\mathcal{P}_1$ and of $\mathcal{P}_2$.
- By the induction hypothesis we obtain winning strategies for $\mathcal{P}_1$ and $\mathcal{P}_2$.
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- This means completeness.

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How do we combine plays?

σ_1	ρ_1, ρ_2	ρ_8, ϕ_1	ρ_2, ϕ_1	ρ_1	ρ_3	ρ_5	ρ_2	
σ_2	ρ_2, ρ_5	ϕ_2, ρ_5	ρ_3, ρ_4	ρ_6, ρ_7	ρ_3, ϕ_2	ρ_9		
σ								

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σ_1	ρ_1, ρ_2	ρ_8, ϕ_1	**stall**	**stall**	ρ_2, ϕ_1	ρ_1	ρ_3	$\cdots$
σ_2	ρ_2, ρ_5	ϕ_2, ρ_5	ρ_3, ρ_4	ρ_6, ρ_7	ρ_3, ϕ_2	ρ_9		
σ	$\rho_1, \rho_2,$ ρ_2, ρ_5	$\rho_8, \phi,$ ϕ, ρ_5	**stall**, ρ_3, ρ_4					

How do we combine plays?

σ_1	ρ_1, ρ_2	ρ_8, ϕ_1	**stall**	**stall**	ρ_2, ϕ_1	ρ_1	ρ_3	$\cdots$
σ_2	ρ_2, ρ_5	ϕ_2, ρ_5	ρ_3, ρ_4	ρ_6, ρ_7	ρ_3, ϕ_2	ρ_9		
σ	$\rho_1, \rho_2,$ ρ_2, ρ_5	$\rho_8, \phi,$ ϕ, ρ_5	**stall**, ρ_3, ρ_4	**stall**, ρ_6, ρ_7				

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σ	$\rho_1, \rho_2,$	$\rho_8, \phi,$	**stall**,	**stall**,	$\rho_2, \phi,$			
	ρ_2, ρ_5	ϕ, ρ_5	ρ_3, ρ_4	ρ_6, ρ_7	ρ_3, ϕ			

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σ_1	ρ_1, ρ_2	ρ_8, ϕ_1	**stall**	**stall**	ρ_2, ϕ_1	ρ_1	ρ_3	$\cdots$
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Soundness and completeness

A proof sketch (cont'd)

Soundness (DLP game semantics $\implies$ Minker semantics)

Similar.

A game semantics for LPN

The **LPNG** game

- Whenever a doubter doubts a negated atom $\sim$p,
  the rôles of the players switch:
  the believer becomes the doubter, doubting p.
- This implies draws.

What about DLPN?

A game semantics seems difficult to define directly.
But *indirectly*. . . ?

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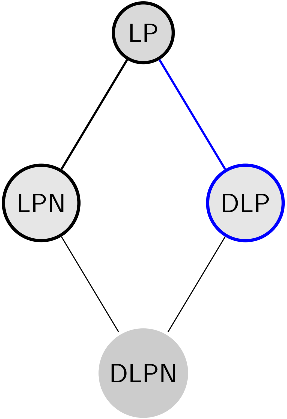

An abstract semantic framework

...and some applications.

Game semantics

- **LP:** Di Cosmo, Loddo & Nicolet (1998)
- **LPN:** Rondogiannis & Wadge (2005)
- **DLP:** me (2013)
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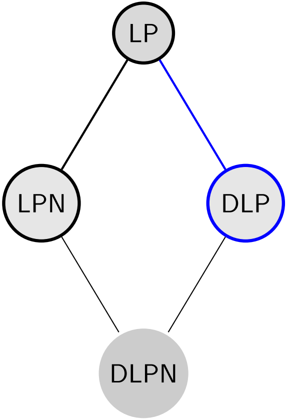

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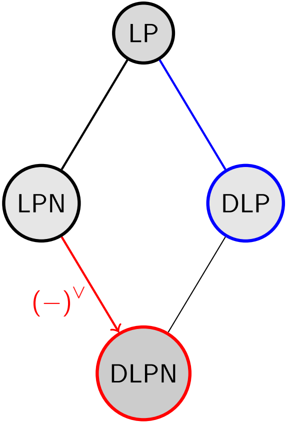

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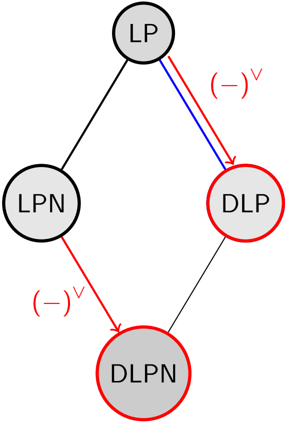

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DLPN: me (2014)

Thanks!

Questions?

Θ.

Bonus tracks. . .

Not is not not

Careful: $\sim$ is *not* $\neg$

$\texttt{a} \leftarrow \sim\texttt{b}$ $\texttt{b} \leftarrow \sim\texttt{a}$ $\texttt{a} \vee \texttt{b} \leftarrow$ all have different meanings.

$\texttt{a} \leftarrow \neg\texttt{b}$ $\texttt{b} \leftarrow \neg\texttt{a}$ $\texttt{a} \vee \texttt{b} \leftarrow$ are equivalent in classical logic.

Why propositional?

Cheating conveniently

Are we forgetting something?

- Logic programs may contain variables and function symbols.
- Instead, we are only looking at (possibly infinite) propositional logic programs.

Why propositional?

Because...

the program

$$\left\{ \begin{array}{r} \mathtt{even}(0) \leftarrow \\ \mathtt{even}(S(S(X))) \leftarrow \mathtt{even}(X) \\ \mathtt{odd}(X) \leftarrow {\sim}\mathtt{even}(X) \end{array} \right\}$$

is semantically equivalent to the program

$$\{\, \mathsf{e}_0 \leftarrow \quad \} \cup \left\{ \begin{array}{c} \mathsf{e}_2 \leftarrow \mathsf{e}_0 \\ \mathsf{e}_4 \leftarrow \mathsf{e}_2 \\ \vdots \end{array} \right\} \cup \left\{ \begin{array}{c} \mathsf{o}_0 \leftarrow {\sim}\mathsf{e}_0 \\ \mathsf{o}_1 \leftarrow {\sim}\mathsf{e}_1 \\ \vdots \end{array} \right\}.$$

Induction?!

But the $\vee$s might be infinite!

Completeness. (Compactness to the rescue!)

Assume G true in every m.m. of $\mathcal{P}$. Then $\mathcal{P} \models G$, and...

$$\begin{aligned} \mathcal{P} \models G &\implies (\exists \mathcal{P}_G \subseteq_{\text{fin}} \mathcal{P})[\mathcal{P}_G \models G] \\ &\implies \exists \text{ winning strategy for } \Gamma_{\mathcal{P}_G}(\leftarrow G) \\ &\qquad \text{...which is also winning for } \Gamma_{\mathcal{P}}(\leftarrow G). \end{aligned}$$

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Soundness.

$\exists$ winning σ for $\Gamma_{\mathcal{P}}(\leftarrow \texttt{G})$ $\implies$ it uses a $\mathcal{P}_\sigma \subseteq_{\text{fin}} \mathcal{P}$
$\implies$ it is also winning in $\Gamma_{\mathcal{P}_\sigma}(\leftarrow \texttt{G})$
$\implies$ G true in every m.m. of $\mathcal{P}_\sigma$
$\implies$ G true in every m.m. of $\mathcal{P} \supseteq \mathcal{P}_\sigma$
(no negation).

Induction?!

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$$\exists \text{ winning } \sigma \text{ for } \Gamma_{\mathcal{P}}(\leftarrow \texttt{G}) \implies \text{it uses a } \mathcal{P}_\sigma \subseteq_{\text{fin}} \mathcal{P}$$

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The induction base

The induction base

- For soundness, we need to make sure that the extra rules "combo" and "implicit", do not enable us to win in any games that we could not already win without them.
- The analogous result for completeness is trivial.

A needed lemma

Lemma (Inclusions)

$$\mathrm{MM}(\mathcal{P}) \subseteq \mathrm{MM}(\mathcal{P}_1) \cup \mathrm{MM}(\mathcal{P}_2) \subseteq \mathrm{HM}(\mathcal{P}).$$

Soundness and completeness

A proof sketch (cont'd)

Soundness (DLP game semantics $\implies$ Minker semantics)

- We have a winning strategy for $\mathcal{P}$.
- We **split this strategy** in two winning strategies, one for $\mathcal{P}_1$ and one for $\mathcal{P}_2$.
- By the induction hypothesis the goal is true in every minimal model of $\mathcal{P}_1$ and of $\mathcal{P}_2$.
- We argue that it must be true in every minimal model of $\mathcal{P}$.
- This means soundness.

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- This means soundness.

How do we split strategies?

Splitting strategies

Whenever the strategy for $\mathcal{P}$ plays the rule ϕ, we play the "restricted" rule ϕ_1 for the game of $\mathcal{P}_1$, or ϕ_2 for $\mathcal{P}_2$.
It is easy to see that this results in a valid, winning strategy.